RADIUS, AREA OF SEGMENT AND CHORD LENGTH

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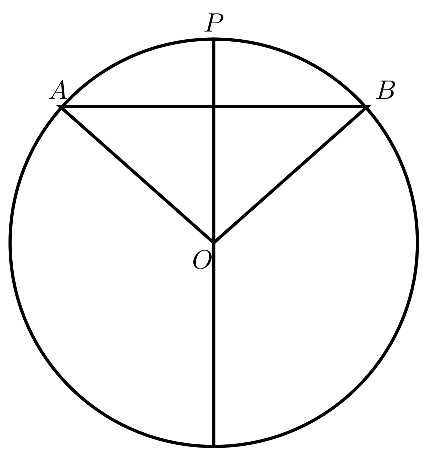
1. Question

A chord of length l divides the area of a circle in the ratio $1 : n$. What is the radius of the circle?
In particular, solve this problem when $l = 8cm$ and $n = 8$.

2. Solution



2.1. **Solution to the general problem.** In the figure, let $\overline{AB}$ be the chord and O, the center. $OA = OB = r$, and $AB = l$.
Area of the segment $ABP = \frac{1}{n+1}$ of area of the circle.

$$A_{seg} = \frac{\pi r^2}{n+1} \tag{1}$$

Let $\angle AOB = \theta$ radians.

$$\begin{aligned} A_{seg} &= \text{Area of sector } OAPB - \text{ Area of } \triangle OAB \\ &= \frac{1}{2}r^2\theta - \frac{1}{2}r^2 \sin\theta \\ &= \frac{r^2(\theta - \sin\theta)}{2} \end{aligned} \tag{2}$$

Combining (1) and (2),

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$$\theta - \sin\theta - \frac{2\pi}{n+1} = 0 \tag{3}$$

where

$$\theta = 2 \cdot \arcsin\frac{l}{2r} \tag{4}$$

That is,

$$r = \frac{l}{2\sin\left(\frac{\theta}{2}\right)} \tag{5}$$

So, the steps involved in solving this problems are
1) Find θ by solving (3).
2) Substitute values in (5) to get r.

2.2. **Solution to the particular problem.** In particular, if $l = 8, n = 8$, (3) becomes

$$\theta - \sin\theta - \frac{2\pi}{9} = 0 \tag{6}$$

which when solved numerically gives $\theta = 1.69092542605$. Substituting in (5), $r = 5.3456$.

2.3. **Other results.**

(1) If the radius r and n are given, the length of chord l can be found by

$$\theta - \sin\theta = \frac{2\pi}{n+1} \tag{7a}$$

$$l = 2r\sin\left(\frac{\theta}{2}\right) \tag{7b}$$

For example, if $r = 21$ and $n = 8$, $\theta = 1.69092542605$, and $l = 31.43$.

(2) If the length of the chord and the radius are given, the ratio of areas is given by

$$\theta = 2 \cdot \arcsin\frac{l}{2r} \tag{8a}$$

$$n = \frac{2\pi}{\theta - \sin\theta} - 1 \tag{8b}$$

For example, if $l = r$, $\theta = \frac{\pi}{3}$, and $n = 33.68$.