# The Hijri calendar

Umesh Nair

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## 1 Introduction

Hijri calendar is followed for religious purposes for Muslims all around the world, and it is the official calendar in a few countries including the Kingdom of Saudi Arabia. It is a pure lunar calendar based on the relative positions of the moon with the sun, as observed from a particular location. Because of this and some other criteria, different variations of the calendar exist. We discuss those in the following sections.

## 2 An example

Table 1 gives the gregorian dates corresponding to the beginning of each month in the year 1429. This Hijri year is contained in the Gregorian year 2008.

Calendars considered are:

1. Values of the astronomical Hijri calendar given in [2]. ICU astronomical calendar matches with this.

2. Values given by ICU's Hijri civil calendar.

3. Microsoft outlook from MS Office 2003.

4. Saudi (Umm al-Quara) Civil date, from http://www.islamicfinder.org/Hcal/index.php

5. Indian calendar, from the printed Malayalam calendar from *Mathrubhoomi* daily.

| Hijri | Astronomical[2] | ICU civil | MS Outlook | Saudi | Indian |
|---|---|---|---|---|---|
| 1429/01/01 | 2008/01/10 | 2008/01/10 | 2008/01/09 | 2008/01/10 | 2008/01/10 |
| 1429/02/01 | 2008/02/08 | 2008/02/09 | 2008/02/08 | 2008/02/08 | 2008/02/09 |
| 1429/03/01 | 2008/03/09 | 2008/03/09 | 2008/03/08 | 2008/03/09 | 2008/03/09 |
| 1429/04/01 | 2008/04/07 | 2008/04/08 | 2008/04/07 | 2008/04/07 | 2008/04/08 |
| 1429/05/01 | 2008/05/07 | 2008/05/07 | 2008/05/06 | 2008/05/06 | 2008/05/07 |
| 1429/06/01 | 2008/06/05 | 2008/06/06 | 2008/06/05 | 2008/06/05 | 2008/06/05 |
| 1429/07/01 | 2008/07/04 | 2008/07/05 | 2008/07/04 | 2008/07/04 | 2008/07/04 |
| 1429/08/01 | 2008/08/03 | 2008/08/04 | 2008/08/03 | 2008/08/02 | 2008/08/02 |
| 1429/09/01 | 2008/09/02 | 2008/09/02 | 2008/09/01 | 2008/09/01 | 2008/09/01 |
| 1429/10/01 | 2008/10/02 | 2008/10/02 | 2008/10/01 | 2008/10/01 | 2008/10/01 |
| 1429/11/01 | 2008/10/31 | 2008/10/31 | 2008/10/30 | 2008/10/30 | 2008/10/30 |
| 1429/12/01 | 2008/11/30 | 2008/11/30 | 2008/11/29 | 2008/11/29 | 2008/11/29 |

Table 1: Gregorian dates for month beginnings of the Hijri year 1429

The dates for two calendars are the same in some months, but differ by a day or two in some other.

# 3    Basic principles

Ideally, a Hijri month starts on the sunset after the new moon, i.e., the first night in which the moon will be up for at least certain time after the sunset.[1]

There are two criteria, specified in Quaran, regarding the start of a new month:

The new month starts at the earliest sunset which occurs

1. *after* the sun-moon conjunction, and

2. *before* the moon sets.

All algorithms try to approximate these astromical criteria. However, in many countries, the new month is declared by a reliegeous person after the actual sighting after sunset following the new moon. If it cannot be sighted, the beginning of the month is on the next day.

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[1]On the next night of the new moon, the moon will be a little above the west horizon at sunset, and will set within 1-2 hours.

# 4 Arithmetic algorithms

## 4.1 Overview

From 9th century CE, reasonably accurate arithmetic calendars were in practice, which is still used in most of the Islamic countries.

A lunar month on average has 29.530589 days. Assuming it is 29.5 days, and the months are set to have 30 and 29 days alternately, we get $29.5 \times 12 = 354$ days. This is 0.367068 days less than $29.530589 \times 12 = 354.367068 \cdots$. Since this is very close to $\frac{11}{30} = 0.3666666 \cdots$, adding 11 leap months every 30 years was considered to be pretty accurate.[2]

So, the odd-numbered months have 30 days while the even-numbered months have 29 days. The twelfth month will have 30 days on leap years which happens 11 times in 30 years.

## 4.2 Leap year variations

The oldest and most popular convention is to use the years 2, 5, 7, 10, 13, 16, 18, 21, 24, 26 and 29 in the 30-year cycle as leap years. Some scholars suggested alternate leap year schemes. The following are the schemes suggested.

1. One popular convention is to use the years 2, 5, 7, 10, 13, 16, 18, 21, 24, 26 and 29 in the 30-year cycle. This is what many books on Arabic calendar, Emacs calendar etc. use. ICU also uses this convention.

2. Another convention has a small variation from the above. It considers the 16th year as non-leap and 15th as leap. The result is, it may have one day difference in some months on two years in a 30-year cycle. MS Outlook Calendar uses this.

3. Yet another convention considers years 2, 5, 8, 10, 13, 16, 19, 21, 24, 27 and 29. Called Fatimid (Misri/Bohra) calendar.

4. Another one with 2, 5, 8, 11, 13, 16, 19, 21, 24, 27 and 30. Not used in modern times.

## 4.3 Epoch variations

Since arithmetic algorithms base on arithmetic calculations rather than the actual astronomical phenomenon, it has to take a base point in time as the starting point (epoch).

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[2]The error is $4 \times 10^{-4}$ per 30 years. This error accumulates to a day in 2492 years.

The calendar started on a sunset on July 15, 622 CE (Julian), and there are two conventions to fix the first day of the calendar.people are still debating whether the previous day or the next day should be considered for arithmetic calendars.

1. The popular approach is to take the next day – i.e., Friday, July 16, 622 CE (in Julian Calendar) – as the first day (1/1/1). This is the called the *Friday epoch.* of the calendar.

2. Another approach is to consider the previous day – Thursday, July 15, 622 CE (Julian) – as the epoch. This is the called the *Thursday epoch.*

Since the calendar is not astronomical, these systems will have one day difference in general.

Because of these variations, there are 8 different arithmetic calendars, not all of which is are used. Table 2 lists these calendars.

| Epoch/Leap scheme | Thursday epoch | Friday epoch |
|---|---|---|
| 1 | *Not used* | ICU, Emacs, Many Books and online calendars |
| 2 | MS Outlook | *Not used* |
| 3 | Fatmid/Bohra | *Not used* |
| 4 | *Not used* | *Not used* |

Table 2: Variations of Hijri calendars

## 4.4 Variations of Arithmetic algorithms

Based on the information in the last section, the following are the variations of arithmetic Hijri calendars.

**The *Standard* Hjri calendar,** used by ICU[3], Emacs Calendar, many online calendars and algorithms given in many books, including *Calendrical Calculations*[3] and *Mapping Time: The calendar and its history*[4]. Uses Leap scheme 1 and Friday epoch. Popular in some Islamic countries.

**The *Kuwaiti* Hijri calendar,** used by Microsoft Outlook[4]. Uses leap scheme 2 and the Thursday epoch. Popular in Kuwait.

**The *Fatmid* "Astronomical" calendar,** used by the Dawoodi Bohras, around 2 million people throughout the world. Uses leap scheme 3, Thursday epoch.

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[3]ICU has support for the Astronomical calendar also.
[4]Microsoft Vista uses the Saudi Umm al-Quara calendar instead

# 5 Astronomical algorithms

In addition to these, there are astronomical calendars. Since they are calculating the occurrence of the astronomical event (i.e., the first night on which the declination of moon is positive), it differs from place to place. In additions to those, there are some variations on how to determine the start of a month. The main variations are given below.

1. In all systems, a month has either 29 or 30 days. Even if the moon is not observed on the $30^{th}$ day, the month will not be extended more than 30 days.

2. In some systems, if the sun-moon conjunction occurs on the $29^{th}$ of a month,

   - If the altitude of the moon is more than some specified angle (usually $2^o$), then the next month starts at that sunset.
   - Otherwise, the next month starts on the next sunset.

   For example, Singapore[1] uses this system[5] taking the reference point as 1°20'34" with altitude limit $2^o$.

3. The Saudi calculations:

   **Before 1395 AH (1975 CE):** Various ways.

   **1395–1419 AH (1975–1998 CE):** When the astronomical new moon occurs less than 3 hours after Saudi midnight (equivalent with 0h UT or Greenwich midnight) the lunar month begins at the previous sunset – otherwise at the sunset following that.

   **1420–1422 AH (1999–2001):** On the 29th of the month, the next day is the first day of the [new] month if the moonset is after sunset at Mecca or else the next day is 30th of the month and the day after is the first day of the new month.

   **Since 1423 AH (2002 CE):** If on the 29th day of the lunar month the two following conditions are satisfied, then the next day is the first day of the new lunar month:

   - The geocentric conjunction occurs before sunset.
   - The moon sets after the sun.

4. Muslims in some countries (India, Pakistan, Bangladesh and the USA[1]) use a true observational system–each month starts when an authority declares it based on actual observation of the new moon.

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[5]The precise definition stipulates that the altitude should be more than $2^o$, the apparant angular distance between the sun and the moon should be more than $3^o$ and the age of the moon should be more than 8 hours.

# 6 Implementation

## 6.1 Arithmetic Algorithms

Arithmetic calendars are straightforward to implement. In general, the conversions from a Hijri date to the fixed offset and back are given in the subsequent sections.

One way to mathematically encapuslate the four schemes described in Section 4.2 is to arrive at a general expression to calculate the number of days elapsed since the beginning of the Hijri epoch to the beginning of a given year.[6]

Since there are 11 leap years in 30 years, it is logical to divide them evenly, and add an extra day when the fraction becaomes a whole number. In other words, the number of extra leap days (in addition to $12 \times 29.5 = 354$ days per year) in $(y-1)$ years is given by

$$n_{leaps}(y) = \left\lfloor \frac{11(y-1)}{30} \right\rfloor \tag{1}$$

But this is assuming that the moon was with strict conjunction with the sun at the beginning of the Hijri era. This is not true. Considering this fact, the above equation can be modified to

$$n_{leaps}(y) = \left\lfloor \frac{11y + \Delta}{30} \right\rfloor \tag{2}$$

where $\Delta$ is a number capturing this variation.

This calculates the number of leap days in $y$ years. It evenly distributes 11 days in 30 years, and the truncated integer value is taken as the number of leap days till the beginning of that year.

Table 3 shows the values for $\Delta = -4, -3, -2, -1, 0, 1, 2, 3, 4$.

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[6][3] gives aan alternate but general theory for all cyclic calendars.

| $\Delta$ | Leap Years | Comments |
|---|---|---|
| -4 | 3 5 8 11 13 16 19 22 24 27 30 | |
| -3 | 2 5 8 11 13 16 19 22 24 27 30 | |
| -2 | 2 5 8 11 13 16 19 21 24 27 30 | Fourth algorithm |
| -1 | 2 5 8 10 13 16 19 21 24 27 30 | |
| 0 | 2 5 8 10 13 16 19 21 24 27 29 | Fatmid algorithm |
| 1 | 2 5 8 10 13 16 18 21 24 27 29 | |
| 2 | 2 5 7 10 13 16 18 21 24 27 29 | |
| 3 | 2 5 7 10 13 16 18 21 24 26 29 | Standard algorithm |
| 4 | 2 5 7 10 13 15 18 21 24 26 29 | Kuwaiti algorithm |

Table 3: Leap year schemes in Hijri calendar

It is clear that the four variations given in Section 4.2 are represented by the values 3, 4, 0 and $-2$ for $\Delta$.

So, the number of days passed since the beginning of the Hijri era is given by

$$d_{epoch}(y) = 354 \times (y-1) + \left\lfloor \frac{11(y-1)+\Delta}{30} \right\rfloor \quad (3)$$

Now, since the months alternates between 30 and 29 days, and leap month occurs only in the end of the year, the number of days passed from the beginning of the year to the beginning of the month is given by

$$d_{year}(m) = \lceil 29.5(m-1) \rceil \quad (4)$$

We can use only integer divisions and avoid floating point operations, by using

$$d_{year}(m) = 29(m-1) + \left\lfloor \frac{m}{2} \right\rfloor \quad (5)$$

Combining all these, the number of days passed since the beginning of the Hijri era to a particular date is given by

$$d_{hepoch}(y,m,d) = 354 \times (y-1) + \left\lfloor \frac{11(y-1)+\Delta}{30} \right\rfloor + \lceil 29.5(m-1) \rceil + d \quad (6)$$

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**Algorithm 1** DAYSSINCEEPOCH(y, m, d, $\Delta$)

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1: **return** $354 \times (y-1) + \left\lfloor \frac{11(y-1)+\Delta}{30} \right\rfloor + 29(m-1) + \left\lfloor \frac{m}{2} \right\rfloor + d$

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Now, let us say the number of days from some epoch to July 15, 622 (Julian) is $\mathcal{H}$. The number of days since this epoch is then $\mathcal{H} + d_{epoch}(y, m, d)$ if we base on the 15th epoch, or $\mathcal{H} + 1 + d_{epoch}(y, m, d)$ if we base on the 16th epoch.

So the final expression is

$$d_{epoch}(y, m, d, \Delta, D_{hepoch}) = \mathcal{H} - 15 + D_{hepoch} + d_{hepoch}(y, m, d) \qquad (7)$$

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**Algorithm 2** FixedFromDate$(y, m, d, \Delta, D_{hepoch})$
---
1: **return** $\mathcal{H} - 15 + D_{hepoch} +$ DaysSinceEpoch$(y, m, d, \Delta)$ /* Alg: 1 */
---

Now, the algorithms for the three arithmetical algorithms given in Section 4.4 are

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**Algorithm 3** FixedFromDateStandard$(y, m, d)$
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1: **return** FixedFromDate$(y, m, d, 3, 16)$ /* Alg: 2 */
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**Algorithm 4** FixedFromDateKuwaiti$(y, m, d)$
---
1: **return** FixedFromDate$(y, m, d, 4, 15)$ /* Alg: 2 */
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**Algorithm 5** FixedFromDateFatmid$(y, m, d)$
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1: **return** FixedFromDate$(y, m, d, 0, 16)$ /* Alg: 2 */
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## 6.2 Astronomical algorithms

TO-DO

# 7 Summary

There are three different arithmetical algorithms, each of which assign a unique Hijri date for each Gregorian date, The dates given by these three algorithms may be different from one another by zero, one or two days.

Also, there are astronomical algorithms, which depends on the location of the reference point. It changed over time, and depends on the location of the reference point.

# Contents

# References

[1] Lasken A. *The Islamic Calendar.* http://www.math.nus.edu.sg/aslasken/calendar/islamic.html.

[2] N. Dershowitz and E. M. Reingold. *Calendrical tabulations 1900-2200.* Cambridge University Press, 2002.

[3] N. Dershowitz and E. M. Reingold. *Calendrical calculations, Third edition.* Cambridge University Press, 2008.

[4] E.G. Richards. *Mapping time, The calendar and its history.* Oxford University Press, 1998.