Analysis of Puzzles

Vol. 1: Simple Puzzles

Umesh Nair

Introduction

Solving puzzles has been my favorite hobby since childhood. Initially, puzzles came in the form of questions and answers. Nobody thought about *how* to find the answer. After learning a little mathematics, I started to solve some of these puzzles by some systematic way. Some were solved by trial and error, but whenever I came across a method by which a puzzle could be solved systematically, I was delighted.

This delight increased when I met smarter people and watched them solve the same puzzles in a different, often more elegant, way. Watching different methods leading to the same solution was a really great thing.

At some point, I started to collect some of the puzzles I came across with *all* the solutions I heard. This is that collection.

I do not know the source of most of these puzzles. I mentioned from where I heard it first, if I remember.

The solver's name is mentioned with most of the solutions. All solutions without a solver's name mentioned are mine. However, this does not mean that they are my *original* solutions. Some of them are; others I read or heard somewhere and just reproduced here.

Another goal of this work is to try to find the most general solution to many popular puzzles for which we normally heard only the particular problem.

The book is divided into three volumes:

Volume 1: Simple Puzzles that can be understood and solved by simple logic and High School Mathematics.

Volume 2: Mathematical Puzzles that require specialized knowledge in some branch of Mathematics.

Volume 3: Programming Puzzles related to computer programming and algorithms.

This is an ongoing effort. Please send your comments to `umesh.p.nair@gmail.com`.

Umesh Nair
Portland, OR, USA.

Contents

Introduction

Solving puzzles has been my favorite hobby since childhood. Initially, puzzles came in the form of questions and answers. Nobody thought about *how* to find the answer. After learning a little mathematics, I started to solve some of these puzzles by some systematic way. Some were solved by trial and error, but whenever I came across a method by which a puzzle could be solved systematically, I was delighted.

This delight increased when I met smarter people and watched them solve the same puzzles in a different, often more elegant, way. Watching different methods leading to the same solution was a really great thing.

At some point, I started to collect some of the puzzles I came across with *all* the solutions I heard. This is that collection.

I do not know the source of most of these puzzles. I mentioned from where I heard it first, if I remember.

The solver's name is mentioned with most of the solutions. All solutions without a solver's name mentioned are mine. However, this does not mean that they are my *original* solutions. Some of them are; others I read or heard somewhere and just reproduced here.

Another goal of this work is to try to find the most general solution to many popular puzzles for which we normally heard only the particular problem.

The book is divided into three volumes:

Volume 1: Simple Puzzles that can be understood and solved by simple logic and High School Mathematics.

Volume 2: Mathematical Puzzles that require specialized knowledge in some branch of Mathematics.

Volume 3: Programming Puzzles related to computer programming and algorithms.

This is an ongoing effort. Please send your comments to `umesh.p.nair@gmail.com`.

Umesh Nair
Portland, OR, USA.

Part I

Questions

Chapter 1

Questions

1.1 Logical

Puzzle 1 Horse race (*Solution : Ch. 2, page 23*)

There are 25 horses, given that only 5 horses can run at a time, so how many races you need to find top 3 horses. You can't measure time but you know the outcomes of the races.

Puzzle 2 Four sons (*Solution : Ch. 3, page 25*)

A farmer had 4 sons $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$. One day the farmer asked them to go to the paddy field and to get the paddy in the bags. All four obeyed the order and after returning from the field each of them gave bags (full of paddy) to their father. It should be noted that they returned at different times and thus none of them was aware of the number of the bags that others had given to their father. Now the farmer called all his sons in the evening and told them that he would give them five clues and they have to tell the number of the bags each of them had brought. The five clues are here :

(i) There were 22 bags.

(ii) Each of them brought different number of bags.

(iii) $\mathcal{A}$ brought the maximum number of bags.

(iv) Number of bags filled by $\mathcal{A}$ is equal to the sum of the number of bags filled by $\mathcal{B}$ and $\mathcal{C}$.

(v) Each son brought at least one bag.

Now, the farmer asked $\mathcal{A}$. But he couldn't answer. Same thing happened for $\mathcal{B}$ and $\mathcal{C}$ also, in order. At last when he asked $\mathcal{D}$, he answered properly.

Can you tell me what is the answer that $\mathcal{D}$ told to his father ? As an additional clue, I can say that the number of the bags that $\mathcal{B}$ has filled is not the minimum of the four.

Puzzle 3 Magic trick (*Solution : Ch. 4, page 33*)

This is a magic trick performed by two magicians, $\mathcal{A}$ and $\mathcal{B}$, with one regular, shuffled deck of 52 cards. $\mathcal{A}$ asks a member of the audience to randomly select 5 cards out of a deck. The audience member – who we will refer to as $\mathcal{C}$ from here on – then hands the 5 cards back to magician $\mathcal{A}$. after looking at the 5 cards, $\mathcal{A}$ picks one of the 5 cards and gives it back to $\mathcal{C}$. $\mathcal{A}$ then arranges the other four cards in some way, and gives those 4 cards face down, in a neat pile, to $\mathcal{B}$. $\mathcal{B}$ looks at these 4 cards and then determines what card is in $\mathcal{C}$'s hand (the missing 5th card).

How is this trick done? There's no secretive message communication in the solution, like encoded speech or hand signals or whatever... The only communication between the two magicians is in the logic of the 4 cards transferred from $\mathcal{A}$ to $\mathcal{B}$.

Think of these magicians as mathematicians.

Puzzle 4 Bolts and doors (*Solution : Ch. 5, page 45*)

You are in a room. On a wall there are two doors. The doors are locked *outside* by 4 horizontal iron bars (called bar1, bar2, bar3 and bar4).

A bar can slide from left to right and right to left between the two doors. A bar always locks one of the two doors. (check ASCII pic below). As they are outside the room, you have no clue of their position.

You can control the bars with buttons. When a bar is activated, it slides from its current door to the other one.

There are 3 buttons (called $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$)

1. Button $\mathcal{A}$ activates a *randomly choosen* bar:
   bar1 OR bar2 OR bar3 OR bar4
2. Button $\mathcal{B}$ activates *randomly* a choice amongst these:
   (bar1 AND bar2) OR (bar2 AND bar3) OR (bar3 AND bar4) OR (bar4 AND bar1)
3. Button $\mathcal{C}$ activates *randomly* a choice amongst these:
   (bar1 AND bar3) OR (bar2 AND bar4))

The problem is:

What is the shortest sequence of button you have to press to be sure you'll be freed ? (that is, to have all bars on either door1 or door2 at some point of the sequence)

ASCII pic: (this is just illustrative, it does not mean the initial configuration is this one)

```
    door1         door2
  ---------      ---------
 |         |   |          |
 |         |   |          |
 |         | <====bar1====>
<====bar2====> |          |
<====bar3====> |          |
 |         | <====bar4====>
 |         |   |          |
_|_________|___|__________|_______
```

Puzzle 5 Cups and Genie (*Solution : Ch. 6, page 49*)

Part 1: There is a round table divided into 4 equal quadrants, with one cup in each quadrant. The quadrants are labeled with letters (A, B, C, D) that do not move. Initially, each cup is randomly face-up or face-down. You are blindfolded and put in front of the table. On each turn of the game, you instruct a genie to flip the cups in whichever positions you choose (e.g., you may say "flip the cups in A and B"), possibly choosing no cups or possibly all four. The genie complies. At this point, if all the cups on the table are face-up, the genie will tell you that you have won the game and are free to go. If not, he rotates the cups randomly (possibly not rotating them) and you play another turn. Give a strategy to win this game in a finite number of moves (the solution is not unique).

Remark: the outcome of "rotation" the four cups is one of the four possible positions: the cups originally at (A,B,C,D) can be at (A,B,C,D), (B,C,D,A), (C,D,A,B), or (D,A,B,C). It is not an arbitrary permutation.

Notice that you cannot examine the current orientation of any cup at any time. This contrasts with the earlier puzzle.

Part 2: Suppose that instead of 4, there are n divisions and cups. For which n is it possible to guarantee a win? Prove your answer is correct.

Puzzle 6 Three saints (*Solution : Ch. 7, page 53*)

There are three identical-looking, infinitely wise and knowledgeable saints - they know everything, but speak only a little. Well, only two words – *wang* and *woong*. One of this means “Yes” and the other “No”, which one is which you don’t know.

One of the saints tells only truth. (he tells the word meaning “yes” when the answer is “yes” and the word meaning “no” when the answer is “no”.)

Another tells only lies. (he tells the word meaning “no” when the answer is “yes” and the word meaning “yes” when the answer is “no”.)

The third one has gone insane during these years, and irrespective of the question asked, he says *wang* or *woong* randomly in answer to any question.

You don't know which saint is which. That is what you need to find.

You can ask the saints three questions altogether. Not necessarily one per each saint. You can ask all the questions to one of them, or you can ask one each, or you can ask one question to one and two questions to another. All questions should result into an "Yes/No" answer. If you ask the same question to two saints, that is counted as two questions. You can decide the next question and the saint to ask depending on the answer of the previous question.

How do you do that?

Note: You need not find which of *wang* and *woong* is "Yes" and which is "No". You need to find which saint is True, False and Random.

Puzzle 7 Four jealous husbands (*Solution : Ch. 8, page 57*)

It is told that four men eloped with their sweethearts, but in carrying out their plan were compelled to cross a stream in a boat which would hold but two persons at a time. In the middle of the stream, there is a small island.

1. It appears that the young men were so extremely jealous that not one of them would permit his prospective bride to remain at any time in the company of any other man or men unless he was also present.
2. Nor was any man to get into a boat alone when there happened to be a girl alone, on the island or shore, other than the one to whom he was engaged.

Let us suppose the river to be two hundred yards wide, with an island in the middle on which any number can stand. How many trips would the boat make to get the four couples safely across in accordance with the imposed conditions?

Puzzle 8 Prisoners' dilemma (*Solution : Ch. 9, page 63*)

The warden meets with the 23 prisoners when they arrive. He tells them:

1. You may meet together today and plan a strategy, but after today you will be in isolated cells and have no communication with one another.
2. There is in this prison a *switch room* which contains two light switches, labelled $\mathcal{A}$ and $\mathcal{B}$, each of which can be in the *on* or *off* position. I am not telling you their present positions. The switches are not connected to any appliance. After today, from time to time, whenever I feel so inclined, I will select one prisoner at random and escort him to

the switch room, and this prisoner will select one of the two switches and reverse its position (e.g. if it was *on*, he will turn it *off*); the prisoner will then be led back to his cell. Nobody else will ever enter the switch room on that day.

3. Each prisoner will visit the switch room aribtrarily often. That is, for any N it is true that eventually each of you will visit the switch room at least N times.)

4. At any time, any of you may declare to me: *We have all visited the switch room.* If it is true (each of the 23 prisoners has visited the switch room at least once), then you will all be set free. If it is false (someone has not yet visited the switch room), you will all remain here forever, with no chance of parole.

Devise for the prisoners a strategy which will guarantee their release.

Puzzle 9 Mule and sugar (*Solution : Ch. 10, page 69*)

Your truck, carrying 10,000 lbs of sugar and a mule, breaks down. You must use the mule to carry the sugar the rest of the way, a 1,000 mile stretch of straight road, to your home. You can load sugar on to the mule to carry, but there are two caveats: a) the mule can't carry more than 1,000 lbs at any given time; and b) the mule will continuously eat the sugar he's carrying, at the rate of 1 lb per mile moved, whenever he's moving, and he will not move, in any direction, if he has no sugar to eat.

Assuming that sugar may only be transported by the mule (you can't carry any, the mule can't pull the truck, and you get no external help), and that you can't prevent the mule from eating the sugar, what is the most amount of the sugar, if any, that can make it to your house?

There are really two problems: deriving a method which yields the maximum amount of sugar at the end of the trip, and reasoning that no other method could yield more.

(*Hint:* At first blush, it might look like the answer is 0: If you put 1,000 lbs on the mule's back and walk straight home, the mule will have just emptied the bag (1 lb/mile x 1000 miles) when you walk in the door of your home.)

Puzzle 10 Red and green disks (*Solution : Ch. 11, page 75*)

You are travelling in the jungles of Africa, when you are caught by a tribe of barbarians. They allow you to choose between death or solving their great challenge.

You are blindfolded and taken to a room, where you are asked to kneel. You feel hundreds of circular discs lying in front of you. You are told that one side of each disc is painted red, and the other, green. There are exactly 129 discs that currently are red side up. You have to divide the discs into two groups, such that each group has the same number of discs showing the red colour. Obviously, no peeking allowed.

Puzzle 11 Subtraction–a game (*Solution : Ch. 12, page 77*)

In the subtraction of one three-digit number from another, Mary and Ann fill in the six digits as follows: Ann chooses a number from 0 through 9, and Mary chooses where to enter it. They continue thus until all blanks are filled. A number may be repeated, and leading zeroes are permitted. Ann tries to make the bottom line (the difference) the greatest and Mary, the smallest. If both players play their best what will be the bottom line?

1.2 Simple Arithmetic

Puzzle 12 Depth of hole (*Solution : Ch. 13, page 81*)

Tom was in the basement playing with the power drill he'd given his son for Christmas, when the the boy went down. *"Could you bore a 5-inch hole with that, dad?"* Ken asked, *"I mean diameter."*.

"Not with this," Tom told him, *"and it would be a big hole."*.

The boy nodded. *"That's what I thought,"* he said. *"But teacher asked how deep the hole would be if we it drilled it centrally through a ball thirteen inches in diameter."*.

Who'd want to do it anyway? But Ken's teacher must have meant a true spherical ball, so how deep would the hole be?

Please give the answer and the method to get the answer.

Puzzle 13 Bee and the trains (*Solution : Ch. 14, page 83*)

This puzzle is often asked with numeric values. I am asking the general question here.

Two trains enter opposite ends of a straight, single track tunnel of length L miles. They have uniform speeds u and v miles per hour. Sitting on the front of one of the trains is a genetically engineered bumble bee. The bee can see in the dark and flies w miles per hour. w is greater than u and v. As soon as the trains enter the tunnel, the bee starts flying back and forth between the front of the two trains. How far will the bee have traveled before the two trains crash to each other?

1.3 Coins and weighing

Puzzle 14 One weighing, 101 coins (*Solution : Ch. 15, page 87*)

There are 101 coins of which exactly 50 coins are artificial. The weight of an artificial coin is just 1 gm. less than that of a real coin. There is a balance which is able to indicate the difference in weights between its two weighning plates. Suppose, you have chosen a coin arbitrarily from those 101 coins. Can you tell us whether that coin is real or artificial by weighing just once ?

1.4 Miscellaneous puzzles

Puzzle 15 General knowledge and arithmetic (*Solution : Ch. 16, page 93*)

1. Start with an unlucky number for a Friday.
2. Multiply by the gables on Hawthorn's house.
3. Add the number a stitch in time saves.
4. Add the number of blind mice.
5. Subtract the number of William Pne du Bois' balloons.
6. Add the number of wonders of the world.
7. Subtract the number of miles in Camptown's racetrack.
8. Subtract the number of strings on a violin.
9. Divide by the number of vertices on a regular hexahedron.
10. Add the number of the engine that ran on Chicago line.
11. Multiply by the number of gentlemen of Verona.
12. Add the atomic number of the element whose symbol is the 25th letter of the alphabet.
13. Divide by the number of hills of Rome.
14. Multiply by the number of railroads on a Monopoly board.
15. Add the number of easy pieces.
16. Add the number of chromosomes in a normal human muscle cell.
17. Multiply by the number of kittens that lost their mittens.
18. Add the number of acres in A. A. Milne's woods.
19. Multiply by the number of cities in Dickens' tale.
20. Subtract the number of degrees Fahrenheit at which Bradbury's books burn.
21. Add the number of Great Lakes.
22. Divide by the number of days of the condor in the title of Grady's book.
23. Subtract the number of blackbirds baked in a pie.

24. Multiply by the number of horsemen of the Apocalypse.
25. Divide by the number of men on a dead man's chest.
26. Add the number of a neutral PH.
27. Subtract the number of carbon atoms in a molecule of ethane.
28. Multiply by the number of heads on Lofting's Pushme-Pullyou.
29. Add the number of miles on the road sign to Juster's Digitopolis.
30. Subtract the number of dried orange pips in a Sherlock Holmes case.
31. Multiply by the number of the square at which Alice met Humpty Dumpty.
32. Divide by the number of witches in Macbeth.
33. Divide by the number of suits in a standard deck of cards.

What is the result?

It is also stated that the final result and all intermediate results are integers.

Part II

Logical puzzles

Chapter 2

Horse race

This is an interesting logic puzzle.*

2.1 Question

There are 25 horses, given that only 5 horses can run at a time, so how many races you need to find top 3 horses. You can't measure time but you know the outcomes of the races.

2.2 Solution

The following are assumed.

1. No two horses are equal. All horses have different speeds.
2. If horse A is faster than horse B in a race, we can assume A is always faster than B. In other words, if A is faster than B, and B is faster than C, A will be faster than C if they are raced together.

Now, the solution:

We need ***seven*** races.

First five races: Divide the 25 horses into five groups A, B, C, D and E, and conduct five races to select the top three in each group. Let them be $A_1 > A_2 > A_3$, $B_1 > B_2 > B_3$, $C_1 > C_2 > C_3$, $D_1 > D_2 > D_3$ and

*Posted by Sharad Kedia in *World of Puzzles* yahoo group.

$E_1 > E_2 > E_3$. No need to consider the rest, because there are at least 3 horses faster than each of them.

Sixth race: Race A_1, B_1, C_1, D_1 and E_1.

WLOG, let us assume that the top three are $A_1 > B_1 > C_1$.

It is clear A_1 is the fastest. So, we need not consider it in the next race.

Now, there is no need to consider

1. any horse in the groups D and E, because A_1, B_1 and C_1 are faster than them.
2. C_2 and C_3, because A_1, B_1 and C_1 are faster than them.
3. B_3, because A_1, B_1 and B_2 are faster than it.

So, we need to consider only A_2, A_3, B_1, B_2 and C_1.

Seventh race: Race A_2, A_3, B_1, B_2 and C_1, and select the top two horses. These, along with A_1 in the first place, are the best three horses.

Chapter 3

Four sons and paddy bags

Pinaki Chakrabarti asked this puzzle in the *MCC June 2002 puzzle contest*.

3.1 Question

A farmer had 4 sons $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$. One day the farmer asked them to go to the paddy field and to get the paddy in the bags. All four obeyed the order and after returning from the field each of them gave bags (full of paddy) to their father. It should be noted that they returned at different times and thus none of them was aware of the number of the bags that others had given to their father. Now the farmer called all his sons in the evening and told them that he would give them five clues and they have to tell the number of the bags each of them had brought. The five clues are here :

(i) There were 22 bags.

(ii) Each of them brought different number of bags.

(iii) $\mathcal{A}$ brought the maximum number of bags.

(iv) Number of bags filled by $\mathcal{A}$ is equal to the sum of the number of bags filled by $\mathcal{B}$ and $\mathcal{C}$.

(v) Each son brought at least one bag. [*]

Now, the farmer asked $\mathcal{A}$. But he couldn't answer. Same thing happened for $\mathcal{B}$ and $\mathcal{C}$ also, in order. At last when he asked $\mathcal{D}$, he answered properly.

[*] This clue was not in the original puzzle statement. This is my addition. See §3.4 on page 28 for a discussion.

Can you tell me what is the answer that $\mathcal{D}$ told to his father ? As an additional clue, I can say that the number of the bags that $\mathcal{B}$ has filled is not the minimum of the four.

3.2 Answer

$\mathcal{A}$ brought 9, $\mathcal{B}$ brought 6, $\mathcal{C}$ brought 3, $\mathcal{D}$ brought 4.

3.3 Solution

This puzzle requires a combination of arithmetic and logic.

3.3.1 Arithmetic part:

Let the sons brought a, b, c and d number of bags. We can assume that each of these numbers is greater than zero and less than 23.

Now, since $a = b + c$, we can say $(2a + d) = 22$, so a and d are related. Since condition (iii) states a is the largest of the four, a should be at least 8. ($a = 7$ will make $d = 8$). Also, a should be at most 10, because $a = 11$ will make $d = 0$.

So, the value of a is one of 8, 9 or 10. And the value of d is 6, 4 or 2 respectively. Also, the four values should be different. The possible combinations are:

```
( C1) ( 8, 7, 1, 6)
( C2) ( 8, 5, 3, 6)
( C3) ( 8, 3, 5, 6)
( C4) ( 8, 1, 7, 6)
( C5) ( 9, 8, 1, 4)
( C6) ( 9, 7, 2, 4)
( C7) ( 9, 6, 3, 4)
( C8) ( 9, 3, 6, 4)
( C9) ( 9, 2, 7, 4)
(C10) ( 9, 1, 8, 4)
(C11) (10, 9, 1, 2)
(C12) (10, 7, 3, 2)
(C13) (10, 6, 4, 2)
(C14) (10, 4, 6, 2)
(C15) (10, 3, 7, 2)
(C16) (10, 1, 9, 2)
```

3.3.2 logic part:

$\mathcal{A}$ and $\mathcal{D}$ knows their own and the other's value. They need to know at least one of b or c to solve the puzzle. $\mathcal{B}$ and $\mathcal{C}$ knows only their own value. They need to know at least one more to solve the puzzle.

$\mathcal{A}$ has 8 in 4 cases, 9 in 6 cases, and 10 in 6 cases. So, $\mathcal{A}$ has no clue whatever his value may be.

$\mathcal{B}$ has 1 in 3 cases, 2 in one case, 3 in 3 cases, 4 in one case, 5 in one case, 6 in 2 cases, 7 in 3 cases, 8 in one case and 9 in one case.

Since $\mathcal{B}$ could not get the values, his value does not belong to a unique case. In other words, his value is not any of 2, 4, 5, 8 and 9. So, the following cases are not possible.

C9, C14, C2, C5, C11.

(C1) (8, 7, 1, 6)
~~(C2) (8, 5, 3, 6)~~
(C3) (8, 3, 5, 6)
(C4) (8, 1, 7, 6)
~~(C5) (9, 8, 1, 4)~~
(C6) (9, 7, 2, 4)
(C7) (9, 6, 3, 4)
(C8) (9, 3, 6, 4)
~~(C9) (9, 2, 7, 4)~~
(C10) (9, 1, 8, 4)
~~(C11) (10, 9, 1, 2)~~
(C12) (10, 7, 3, 2)
(C13) (10, 6, 4, 2)
~~(C14) (10, 4, 6, 2)~~
(C15) (10, 3, 7, 2)
(C16) (10, 1, 9, 2)

So, when the problem is presented to C, only 11 cases are possible:

```
( C1) ( 8, 7, 1, 6)
( C3) ( 8, 3, 5, 6)
( C4) ( 8, 1, 7, 6)
( C6) ( 9, 7, 2, 4)
( C7) ( 9, 6, 3, 4)
( C8) ( 9, 3, 6, 4)
(C10) ( 9, 1, 8, 4)
(C12) (10, 7, 3, 2)
(C13) (10, 6, 4, 2)
(C15) (10, 3, 7, 2)
```

```
(C16) (10, 1, 9, 2)
```

Here $\mathcal{C}$ has 1 in one case, 2 in one case, 3 in 2 cases, 4 in one case, 5 in one case, 6 in one case, 7 in two cases, 8 in one case, and 9 in one case. All one-case cases can be eliminated, because $\mathcal{C}$ could not answer. So, the remaining cases are:

~~(C1) (8, 7, 1, 6)~~
~~(C3) (8, 3, 5, 6)~~
(C4) (8, 1, 7, 6)
~~(C6) (9, 7, 2, 4)~~
(C7) (9, 6, 3, 4)
~~(C8) (9, 3, 6, 4)~~
~~(C10) (9, 1, 8, 4)~~
(C12) (10, 7, 3, 2)
~~(C13) (10, 6, 4, 2)~~
(C15) (10, 3, 7, 2)
~~(C16) (10, 1, 9, 2)~~

Which means the following four cases:

```
( C4) ( 8, 1, 7, 6)
( C7) ( 9, 6, 3, 4)
(C12) (10, 7, 3, 2)
(C15) (10, 3, 7, 2)
```

Now, $\mathcal{D}$ *could* answer. That means $d \neq 2$, because if d were 2, he could not have identified which of the two cases (C12) or (C15).

Now, the case must be either (C4) or (C7). $\mathcal{D}$ doesn't have confusion, because he knows his value, but we do. The final clue (given to us, not to them) that $\mathcal{B}$ doesn't have the minimum value eliminates (C4). So, the answer is (C7).

So,

- $\mathcal{A}$ brought 9,
- $\mathcal{B}$ brought 6,
- $\mathcal{C}$ brought 3, and
- $\mathcal{D}$ brought 4.

3.4 Tries, traps and pitfalls

1. *Why is the assumption that each son brought at least one bag is important?*

We know from the final answer that everyone brought at least one bag. However, it is important that it is explicitely stated *and* everyone knew it; otherwise, the puzzle is insolvable. Let us see what happens if one of them *could* bring zero bags.

It is clear that only $\mathcal{D}$ can have zero, (otherwise, other conditions will be violated), but it will introduce the following cases:

```
(C17) (11, 1, 10, 0)
(C18) (11, 2, 9, 0)
(C19) (11, 3, 8, 0)
(C20)(11, 4, 7, 0)
(C21) (11, 5, 6, 0)
(C22) (11, 6, 5, 0)
(C23) (11, 7, 4, 0)
(C24) (11, 8, 3, 0)
(C25) (11, 9, 2, 0)
(C26) (11, 10, 1, 0)
```

These cases represent all values for $\mathcal{B}$ and C. As a result, no case (in the original set) can be eliminated by A, $\mathcal{B}$ or C, and the entire set is presented to D. $\mathcal{D}$ will be completely clueless to answer.

However, one can argue: "Hey, the puzzle states that each of them gave *bags* to their father, so we can assume that each gave at least one, and everyone knows that". Right, but in that case, each should have brought at least *two* (because of *bags*, which is plural), thus eliminating (C1), (C4), (C5), (C10), (C11) and (C16) also in the beginning. Let us try again with it:

$\mathcal{B}$ gets the following, and eliminates its unique values:

~~(C2) (8, 5, 3, 6)~~
(C3) (8, 3, 5, 6)
(C6) (9, 7, 2, 4)
(C7) (9, 6, 3, 4)
(C8) (9, 3, 6, 4)
~~(C9) (9, 2, 7, 4)~~
(C12) (10, 7, 3, 2)
(C13) (10, 6, 4, 2)
~~(C14) (10, 4, 6, 2)~~
(C15) (10, 3, 7, 2)

Now, $\mathcal{C}$ eliminates his own unique values:

~~(C3) (8, 3, 5, 6)~~
~~(C6) (9, 7, 2, 4)~~

```
( C7) ( 9, 6, 3, 4)
~~( C8) ( 9, 3, 6, 4)~~
(C12) (10, 7, 3, 2)
~~(C13) (10, 6, 4, 2)~~
~~(C15) (10, 3, 7, 2)~~
```

leaving this to $\mathcal{D}$.

```
( C7) ( 9, 6, 3, 4)
(C12) (10, 7, 3, 2)
```

$\mathcal{D}$ could answer this, but we cannot. So, this is indeterminate.

So, depending on the grammar is not right. The question is what does "each of them gave *bags* to their father" mean?

Each of them brought at least one. This is assumed to solve the puzzle. And, it is assumed that *they* knew this.

Each of them brought at least two. This is fine, for the final answer. But it is not clear whether they knew this. If they did, this puzzle is insolvable.

Nothing about the number each brought. Can be anything from 0 to 22, subject to other conditions. But, allowing 0 makes the problem insolvable.

That is why that assumption is explicitely stated.

3.4.1 What is the most dangerous pitfall in this puzzle?

Assuming that the final clue (that $\mathcal{B}$ didn't bring the minimum number of bags) is known to everybody. This will eliminate more cases too early, making the puzzle insolvable. See below:

After the arithmetic analysis, we get the following combinations:

```
( C1) ( 8, 7, 1, 6)
( C2) ( 8, 5, 3, 6)
( C3) ( 8, 3, 5, 6)
( C4) ( 8, 1, 7, 6)
( C5) ( 9, 8, 1, 4)
( C6) ( 9, 7, 2, 4)
( C7) ( 9, 6, 3, 4)
( C8) ( 9, 3, 6, 4)
( C9) ( 9, 2, 7, 4)
```

(C10) (9, 1, 8, 4)
(C11) (10, 9, 1, 2)
(C12) (10, 7, 3, 2)
(C13) (10, 6, 4, 2)
(C14) (10, 4, 6, 2)
(C15) (10, 3, 7, 2)
(C16) (10, 1, 9, 2)

If they knew that $\mathcal{B}$ didn't bring the least amount, they can eliminate a few cases:

(C1) (8, 7, 1, 6)
(C2) (8, 5, 3, 6)
~~(C3) (8, 3, 5, 6)~~
~~(C4) (8, 1, 7, 6)~~
(C5) (9, 8, 1, 4)
(C6) (9, 7, 2, 4)
(C7) (9, 6, 3, 4)
~~(C8) (9, 3, 6, 4)~~
~~(C9) (9, 2, 7, 4)~~
~~(C10) (9, 1, 8, 4)~~
(C11) (10, 9, 1, 2)
(C12) (10, 7, 3, 2)
(C13) (10, 6, 4, 2)
(C14) (10, 4, 6, 2)
(C15) (10, 3, 7, 2)
~~(C16) (10, 1, 9, 2)~~

$\mathcal{A}$ is still clueless, but $\mathcal{B}$ will get the following:

(C1) (8, 7, 1, 6)
(C2) (8, 5, 3, 6)
(C5) (9, 8, 1, 4)
(C6) (9, 7, 2, 4)
(C7) (9, 6, 3, 4)
(C11) (10, 9, 1, 2)
(C12) (10, 7, 3, 2)
(C13) (10, 6, 4, 2)
(C14) (10, 4, 6, 2)
(C15) (10, 3, 7, 2)

Now removing cases where $\mathcal{B}$ has only one case:

(C1) (8, 7, 1, 6)
~~(C2) (8, 5, 3, 6)~~

```
~~( C5) ( 9, 8, 1, 4)~~
( C6) ( 9, 7, 2, 4)
( C7) ( 9, 6, 3, 4)
~~(C11) (10, 9, 1, 2)~~
(C12) (10, 7, 3, 2)
(C13) (10, 6, 4, 2)
~~(C14) (10, 4, 6, 2)~~
~~(C15) (10, 3, 7, 2)~~
```

This is what is presented to C.

```
( C1) ( 8, 7, 1, 6)
( C6) ( 9, 7, 2, 4)
( C7) ( 9, 6, 3, 4)
(C12) (10, 7, 3, 2)
(C13) (10, 6, 4, 2)
```

Eliminating the cases where $\mathcal{C}$ has only one case, it becomes:

```
~~( C1) ( 8, 7, 1, 6)~~
~~( C6) ( 9, 7, 2, 4)~~
( C7) ( 9, 6, 3, 4)
(C12) (10, 7, 3, 2)
~~(C13) (10, 6, 4, 2)~~
```

$\mathcal{D}$ *could* answer, because he knows how many he has. But *we* do not know that. We are stuck here.

Many people proceded this way, and after getting these two values, they interpreted that the meaning of *As an additional clue, I can say that the number of the bags that $\mathcal{B}$ has filled is not the minimum* in the puzzle as *you will get two answers, and discard the one where $\mathcal{B}$ has the minimum of the two.*, and gave (10, 7, 3, 2) as the answer. That way, it can be considered a *try*.

Chapter 4

Magic puzzle

This puzzle[*] analyzes a card trick performed by magicians. Two solutions–a more practical particular solution and a more general but difficult to practise solution–are given.

4.1 Question

This is a magic trick performed by two magicians, $\mathcal{A}$ and $\mathcal{B}$, with one regular, shuffled deck of 52 cards. $\mathcal{A}$ asks a member of the audience to randomly select 5 cards out of a deck. The audience member – who we will refer to as $\mathcal{C}$ from here on – then hands the 5 cards back to magician $\mathcal{A}$. after looking at the 5 cards, $\mathcal{A}$ picks one of the 5 cards and gives it back to $\mathcal{C}$. $\mathcal{A}$ then arranges the other four cards in some way, and gives those 4 cards face down, in a neat pile, to $\mathcal{B}$. $\mathcal{B}$ looks at these 4 cards and then determines what card is in $\mathcal{C}$'s hand (the missing 5th card).

How is this trick done? There's no secretive message communication in the solution, like encoded speech or hand signals or whatever... The only communication between the two magicians is in the logic of the 4 cards transferred from $\mathcal{A}$ to $\mathcal{B}$.

Think of these magicians as mathematicians.

4.2 Test cases

If you find a solution, test it with the following test cases:

[*]Posted by Sharad Kedia in *World of Puzzles* yahoo group.

1. $\clubsuit 2$, $\clubsuit J$, $\diamondsuit 7$, $\heartsuit Q$, and $\spadesuit Q$.
2. $\diamondsuit 8$, $\heartsuit 2$, $\heartsuit 4$, $\heartsuit 7$ and $\heartsuit K$.
3. $\clubsuit A$, $\clubsuit 9$, $\diamondsuit 2$, $\heartsuit 1$, and $\spadesuit K$.
4. $\clubsuit A$, $\clubsuit 9$, $\diamondsuit 2$, $\heartsuit 2$, and $\spadesuit K$.

4.3 Definitions

The following definitions may help in understanding certain points in the solutions discussed:

- The *circular difference* $\mathcal{D}_n(a,b)$ between two integers a and b in a finite consecutive set [1 . . . n] is given by

$$\mathcal{D}_n(a,b) = (a-b) \mod n \tag{4.1}$$

  In other words,

$$\mathcal{D}_n(a,b) = \begin{cases} a-b & \text{if } b \leq a \\ a+n-b & \text{otherwise} \end{cases} \tag{4.2}$$

  For example, $\mathcal{D}_{15}(7,3) = 4$, while $\mathcal{D}_{15}(3,7) = 11$.

  It is easy to note that $\mathcal{D}_n(a,b) + \mathcal{D}_n(b,a) = n$.

- The *circular sum* $\mathcal{S}_n(a,b)$ between two integers a and b in a finite consecutive set [1 . . . n] is given by

$$\mathcal{S}_n(a,b) = ((a+b-1) \mod n) + 1 \tag{4.3}$$

  In other words,

$$\mathcal{S}_n(a,b) = \begin{cases} a+b & \text{if } (a+b) \leq n \\ a+b-n & \text{otherwise} \end{cases} \tag{4.4}$$

  For example, $\mathcal{S}_{15}(8,7) = 15$, while $\mathcal{S}_{15}(11,7) = 3$.

4.4 Sorting Order

In order to arrive at a solution, we need to define a sorting order for the 52 cards. Any well-defined order can be used. For illustrating the solutions, the order given in table 4.1 on the next page is used. The sorting order uses suits first and the ranks next, with $\clubsuit < \diamondsuit < \heartsuit < \spadesuit$ and $A < 1 < 2 < \ldots < 10 < J < Q < K$, so that $\clubsuit A < 1\clubsuit < \ldots \spadesuit Q < \spadesuit K$.

$\clubsuit A$	$\rightarrow 1$	$\diamondsuit A$	$\rightarrow 14$	$\heartsuit A$	$\rightarrow 27$	$\spadesuit A$	$\rightarrow 40$
$\clubsuit 2$	$\rightarrow 2$	$\diamondsuit 2$	$\rightarrow 15$	$\heartsuit 2$	$\rightarrow 28$	$\spadesuit 2$	$\rightarrow 41$
$\clubsuit 3$	$\rightarrow 3$	$\diamondsuit 3$	$\rightarrow 16$	$\heartsuit 3$	$\rightarrow 29$	$\spadesuit 3$	$\rightarrow 42$
$\clubsuit 4$	$\rightarrow 4$	$\diamondsuit 4$	$\rightarrow 17$	$\heartsuit 4$	$\rightarrow 30$	$\spadesuit 4$	$\rightarrow 43$
$\clubsuit 5$	$\rightarrow 5$	$\diamondsuit 5$	$\rightarrow 18$	$\heartsuit 5$	$\rightarrow 31$	$\spadesuit 5$	$\rightarrow 44$
$\clubsuit 6$	$\rightarrow 6$	$\diamondsuit 6$	$\rightarrow 19$	$\heartsuit 6$	$\rightarrow 32$	$\spadesuit 6$	$\rightarrow 45$
$\clubsuit 7$	$\rightarrow 7$	$\diamondsuit 7$	$\rightarrow 20$	$\heartsuit 7$	$\rightarrow 33$	$\spadesuit 7$	$\rightarrow 46$
$\clubsuit 8$	$\rightarrow 8$	$\diamondsuit 8$	$\rightarrow 21$	$\heartsuit 8$	$\rightarrow 34$	$\spadesuit 8$	$\rightarrow 47$
$\clubsuit 9$	$\rightarrow 9$	$\diamondsuit 9$	$\rightarrow 22$	$\heartsuit 9$	$\rightarrow 35$	$\spadesuit 9$	$\rightarrow 48$
$\clubsuit 10$	$\rightarrow 10$	$\diamondsuit 10$	$\rightarrow 23$	$\heartsuit 10$	$\rightarrow 36$	$\spadesuit 10$	$\rightarrow 49$
$\clubsuit J$	$\rightarrow 11$	$\diamondsuit J$	$\rightarrow 24$	$\heartsuit J$	$\rightarrow 37$	$\spadesuit J$	$\rightarrow 50$
$\clubsuit Q$	$\rightarrow 12$	$\diamondsuit Q$	$\rightarrow 25$	$\heartsuit Q$	$\rightarrow 38$	$\spadesuit Q$	$\rightarrow 51$
$\clubsuit K$	$\rightarrow 13$	$\diamondsuit K$	$\rightarrow 26$	$\heartsuit K$	$\rightarrow 39$	$\spadesuit K$	$\rightarrow 52$

Table 4.1: A sorting order of cards

4.5 Solution

4.5.1 Solver's description

There are 4 cards, and you need to indicate one of the 48 remaining cards. You can do only 4! = 24 distinct cases with permutations of four cards. Perhaps $\mathcal{A}$ can give $\mathcal{B}$ face-up or face-down to get the extra information. Or he can use a deck of "magic" cards with a non-symmetric picture on the other side and use the orientation. But the question says "normal cards" and "face down" explicitely, ruling out all these possibilities. So, there must be another way.

Now, since there are 5 cards, $\mathcal{A}$ always can find two cards of the same suit. Since he can choose the card to be given to $\mathcal{C}$, he can choose one of these two cards, and place the other card as the first card to indicate the suit. Now, we have three cards to denote 13 ranks, which is impossible, because permutations of 3 cards would show only 3! = 6 cases. Duh!

So, in addition to showing the suit this way, we need to use the first card also to indicate the rank of the card. We can represent the *difference* between the first card and the $\mathcal{C}$-card.

However, the difference is at least 1 (the two cards cannot be the same) or at most 12 (Consider an Ace and a King, if the numbers 1-13 represent A-1-2-3-4-5-6-7-8-9-10-J-Q-K). However, we can use a *circular difference*, where Ace comes after King (as in some Rummy games), so that Ace = King + 1. Mathematically, we can find $\mathcal{D}_{13}(r_1, r_2)$. Now, there are two differences between two ranks, found in either direction (For example, difference between 4 and J is 11-4 = 7 or 13+4-11 = 6 in the other direction), and these two *differences* add to 13. That means one of these differences should be less than or equal to 6, which we can represent using three remaining cards. For doing that we can use

Cards	Number
LMH	1
LHM	2
MLH	3
MHL	4
HLM	5
HML	6

Table 4.2: An aribitary rule for ordering three cards

some arbitary ordering of the four suites (for example, $\clubsuit\diamondsuit\heartsuit\spadesuit$, as in Bridge).

We can use any arbitary rule, one example is given in table 4.2. Let us denote the three remaining cards as L (Low), M (Middle), H (High) based on the suit and rank of them (in any order, let us say rank first and suit next)

Done!

4.5.2 A working algorithm

A working procedure, implemented based on the description in the previous section, is described in algorithms 1 and 2.

Algorithm 1 gives an algorithm to be followed by $\mathcal{A}$ to perform this trick, and algorithm 2 on the next page gives the corresponding algorithm for $\mathcal{B}$.

Algorithm 1 Solution 1 : $\mathcal{A}$'s procedure

Sort the cards in ascending order.
Find the smallest pair with the same suit. Let it be a and b.
Find the circular difference $k = \mathcal{D}_{13}(Rank_b, Rank_a)$.
if $k \leq 6$ **then**
    Give b to $\mathcal{C}$.
    Put a as first card.
else
    Give a to $\mathcal{C}$.
    $k = \mathcal{D}_{13}(Rank_b, Rank_a)$.
    Put b as first card.
end if
Find the order corresponding to k from table 4.2.
Arrange the remaining three cards in that order

Now, the testcases:

Algorithm 2 Solution 1 : $\mathcal{B}$'s procedure

1: Check the first card. Let $s \Leftarrow \{\text{suit of the card}\}$, $r \Leftarrow \{\text{rank of the card}\}$.
2: inspect the remaining three cards, and assuming the sorting order mentioned above, name the cards H (high), M (middle) and L (low).
3: Find the actual order of these cards. Get the corresponding number from table 4.2 on the preceding page. Let it be n.
4: Find $k \Leftarrow \mathcal{S}_{13}(r, n)$.
5: The card given to $\mathcal{C}$ has rank k and suit s.

4.5.3 Test case 1

$\mathcal{A}$ proceeds as follows:

2: $a = \clubsuit 2$, $b = \clubsuit J$.
3: $k = 11 - 2 = 9$.
4: False
8: Give $\clubsuit 2$ to $\mathcal{C}$.
9: $k = 4$.
10: Put $\clubsuit J$ as first card.
12: Order for 4 is MHL.
13: Arrange $\diamondsuit 7, \spadesuit Q, \heartsuit Q$ in the order $\spadesuit Q, \heartsuit Q, \diamondsuit 7$, to match MHL.
14: Give $\mathcal{B}$ the order $\clubsuit J, \spadesuit Q, \heartsuit Q, \diamondsuit 7$.

$\clubsuit 2$
$\clubsuit J$
$\diamondsuit 7$
$\heartsuit Q$
$\spadesuit Q$

So, $\mathcal{A}$ gives $\clubsuit 2$ to $\mathcal{C}$, and gives $\clubsuit J - \spadesuit Q - \heartsuit Q - \diamondsuit 7$ to $\mathcal{B}$.

$\mathcal{B}$ proceeds as follows:

1: $s = \clubsuit, r = 11$.
2: H $= \spadesuit Q$, M $= \heartsuit Q$, L $= \diamondsuit 7$.
3: Order is MHL, so $n = 4$.
4: $k = 11 + 4 - 13 = 2$.
5: So the missing card is $\clubsuit 2$.

4.5.4 Test case 2

$\mathcal{A}$ proceeds as follows:

2: $a = \heartsuit 2$, $b = \heartsuit 4$.
3: $k = 4 - 2 = 2$.
4: True
5: Give $b = \heartsuit 4$ to $\mathcal{C}$.
6: Put $\heartsuit 2$ as first card.
12: Order for 4 is LHM.
13: Arrange $\diamondsuit 8, \heartsuit 7, \heartsuit K$ in the order $\diamondsuit 8, \heartsuit K, \heartsuit 7$, to match LHM.
14: Give $\mathcal{B}$ the order $\heartsuit 2$, $\diamondsuit 8, \heartsuit K, \heartsuit 7$.

$\diamondsuit 8$
$\heartsuit 2$
$\heartsuit 4$
$\heartsuit 7$
$\heartsuit K$

So, $\mathcal{A}$ gives $\heartsuit 4$ to $\mathcal{C}$, and gives $\heartsuit 2 - \diamondsuit 8 - \heartsuit K - \heartsuit 7$ to $\mathcal{B}$.

$\mathcal{B}$ proceeds as follows:

1: $s = \heartsuit, r = 2$.
2: H = $\heartsuit K$, M = $\heartsuit 7$, L = $\diamondsuit 8$.
3: Order is LHM, so $n = 2$.
4: $k = 2 + 2 = 4$.
5: So the missing card is $\heartsuit 4$.

4.5.5 Test case 3

$\mathcal{A}$ proceeds as follows:

2: $a = \clubsuit A$, $b = \clubsuit 9$.
3: $k = 9 - 1 = 8$.
4: False
8: Give $a = \clubsuit A$ to $\mathcal{C}$.
9: $k = 13 + 1 - 9 = 5$.
10: Put $\clubsuit 9$ as first card.
12: Order for 5 is HLM.
13: Arrange $\diamondsuit 2, \heartsuit 1, \spadesuit K$ in the order $\spadesuit K, \diamondsuit 2, \heartsuit 1$, to match HLM.

$\clubsuit A$
$\clubsuit 9$
$\diamondsuit 2$
$\heartsuit 1$
$\spadesuit K$

So, $\mathcal{A}$ gives $\clubsuit A$ to $\mathcal{C}$, and gives $\clubsuit 9 - \spadesuit K - \diamondsuit 2 - \heartsuit 1$ to $\mathcal{B}$.

$\mathcal{B}$ proceeds as follows:

1: $s = \clubsuit, r = 9$.
2: H = $\spadesuit K$, M = $\heartsuit 1$, L = $\diamondsuit 2$.
3: Order is HLM, so $n = 5$.
4: $k = 9 + 5 - 13 = 1$.
5: So the missing card is $\clubsuit A$.

4.5.6 Test case 4

$\mathcal{A}$ proceeds as follows:

2: $a = \clubsuit A$, $b = \clubsuit 9$.
3: $k = 9 - 1 = 8$.
4: False
8: Give $a = \clubsuit A$ to $\mathcal{C}$.
9: $k = 13 + 1 - 9 = 5$.
10: Put $\clubsuit 9$ as first card.
12: Order for 5 is HLM.
13: Arrange $\diamondsuit 2, \heartsuit 2, \spadesuit K$ in the order $\spadesuit K, \diamondsuit 2, \heartsuit 2$, to match HLM.

$\clubsuit A$
$\clubsuit 9$
$\diamondsuit 2$
$\heartsuit 2$
$\spadesuit K$

So, $\mathcal{A}$ gives $\clubsuit A$ to $\mathcal{C}$, and gives $\clubsuit 9 - \spadesuit K - \diamondsuit 2 - \heartsuit 2$ to $\mathcal{B}$.

$\mathcal{B}$ proceeds as follows:

1: $s = \clubsuit, r = 9$.
2: H = $\spadesuit K$, M = $\heartsuit 2$, L = $\diamondsuit 2$.
3: Order is HLM, so $n = 5$.
4: $k = 9 + 5 - 13 = 1$.
5: So the missing card is $\clubsuit A$.

4.6 Solution 2

This is a less practical but more general solution*.

4.6.1 Solver's description

This assumes that the two magicians previously agrees to some conventions for the following two:

1. Each card is given a number from 1 to 52.

   This can be any numering scheme. For illustration, values given in table 4.1 on page 35 is used.

2. The order of four unequal numbers (a, b, c, d in the ascending order) represent a number from 1 to 24.

   Any rule can be used here also. For illustration, the rule given in table 4.3 is used.

abcd	→ 1	bcad	→ 9	cdba	→ 17
abdc	→ 2	bcda	→ 10	cdab	→ 18
acbd	→ 3	bdac	→ 11	dbca	→ 19
acdb	→ 4	bdca	→ 12	dbac	→ 20
adbc	→ 5	cbad	→ 13	dcba	→ 21
adcb	→ 6	cbda	→ 14	dcab	→ 22
bacd	→ 7	cabd	→ 15	dabc	→ 23
badc	→ 8	cadb	→ 16	dacb	→ 24

Table 4.3: An aribitary rule for ordering four cards

$\mathcal{A}$ takes the cards given by $\mathcal{C}$ and arranges those cards in the ascending order (based on the numbering system decided earlier). Lets say those cards as...

$$x_1, x_2, x_3, x_4, x_5$$

Now, there are two cases:

*Due to Vamsi Reddy.

Case 1 : $(x_5 - x_4) \leq 24$

$\mathcal{A}$ gives x_5 to $\mathcal{C}$, and some arrangement of x_1, x_2, x_3, x_4 to $\mathcal{B}$. Using the order of these four cards, he can represent a number from 1 to 24. Let it be n.

$\mathcal{B}$ recognizes the fifth card as the one represented by the number

$$(x_4 + n) \mod 49^*$$

Case 2 : $(x_5 - x_4) > 24$

otherwise, $\mathcal{A}$ gives x_1 to $\mathcal{A}$, and some arrangement of x_2, x_3, x_4, x_5 to $\mathcal{B}$, representing a number from 1 to 24. Let it be n.

$\mathcal{B}$ recognizes the fifth card as the one represented by the number

$$(x_5 + n) \mod 49$$

In either case, $\mathcal{B}$ picks the highest card (x_h) and the fifth card is

$$(x_h + n) \mod 49$$

4.6.2 A working algorithm

The method given in the previous section is slighly modified in algorithms 3 and 4. Vamsi's solution used arithmetic modulo 49 (in fact, it should be 48, with the range $0 - 47$ adjusted to $1 - 48$), and *leaping over* the four cards $\mathcal{B}$ already knows while doing the arithmetic. My modified version uses arithmetic modulo 52, so that the computation is easier.

Algorithm 3 on the next page gives an algorithm to be followed by $\mathcal{A}$ to perform this trick, and algorithm 4 on the next page gives the corresponding algorithm for $\mathcal{B}$.

Now, the testcases:

4.6.3 Test case 1

$\mathcal{A}$ proceeds as follows:

---

*In fact, this should be $((x_4 + n) \mod 48) + 1$, and the addition should be performed by *leaping over* the four cards $\mathcal{B}$ already know. This is true for the two additions given in *Case 2* as well. See §4.6.2 on page 40 for a modified version.

Algorithm 3 Solution 2 : $\mathcal{A}$'s procedure

Sort the cards in ascending order. Let the numbers correponding to the cards are x_1, x_2, x_3, x_4 and x_5, with x_1 being the smallest and x_5 being the largest.
Find $d \Leftarrow x_5 - x_4$.
if $d \leq 24$ **then**
    Give the card corresponding to x_5 to $\mathcal{C}$.
    From table 4.3 on page 39, find the order corresponding to d, and arrange x_1, x_2, x_3 and x_4, considering $a \leftarrow x_1$, $b \leftarrow x_2$, $c \leftarrow x_3$ and $d \leftarrow x_4$.
else
    Give the card corresponding to x_1 to $\mathcal{C}$.
    $k \Leftarrow \mathcal{D}_{52}(x_1, x_5)$. Now $k \leq 24$.
    From table 4.3 on page 39, find the order corresponding to k, and arrange x_2, x_3, x_4 and x_5, considering $a \leftarrow x_2$, $b \leftarrow x_3$, $c \leftarrow x_4$ and $d \leftarrow x_5$.
end if
Give the four cards with the above order to $\mathcal{B}$.

Algorithm 4 Solution 2 : $\mathcal{B}$'s procedure

Look at the highest card according to table 4.1 on page 35 (it may be at any of the four position), and note its number. Let it be h.
Check the order of the four cards, and find the number corresponding to it from table 4.3 on page 39, with a, b, c, d corrsponding to the smallest to the largest card. Let it be n.
Find $k \Leftarrow \mathcal{S}_{52}(h, n)$.
Find the card corresponding to k from table 4.1 on page 35. This is the card given to $\mathcal{C}$.

1: Numbers for the cards, arranged in ascending order, are $x_1 = 2, x_2 = 11, x_3 = 20, x_4 = 38$ and $x_5 = 51$.
2: $d = 51 - 38 = 13$.
3: True
4: Give $x_5(\spadesuit Q)$ to $\mathcal{C}$.
5: The order for 13 is *cbad*, So the order is $\diamondsuit 7 - \clubsuit J - \clubsuit 2 - \heartsuit Q$.

$\clubsuit 2$
$\clubsuit J$
$\diamondsuit 7$
$\heartsuit Q$
$\spadesuit Q$

So, $\mathcal{A}$ gives $\spadesuit Q$ to $\mathcal{C}$, and gives $\diamondsuit 7 - \clubsuit J - \clubsuit 2 - \heartsuit Q$ to $\mathcal{B}$.

$\mathcal{B}$ proceeds as follows:

1: Highest card $= \heartsuit Q$. $h = 38$.
2: The order is *cbad*. So, $n = 13$.
3: $k = 38 + 13 = 51$.
5: So the missing card is $\spadesuit Q$.

4.6.4 Test case 2

$\mathcal{A}$ proceeds as follows:

1: Numbers for the cards, arranged in ascending order, are $x_1 = 21, x_2 = 28, x_3 = 30, x_4 = 33$ and $x_5 = 39$.
2: $d = 39 - 33 = 6$.
3: True
4: Give $x_5(\heartsuit K)$ to $\mathcal{C}$.
9: The order for 6 is *adcb*. So the order is $\diamondsuit 8 - \heartsuit 7 - \heartsuit 4 - \heartsuit 2$.

$\clubsuit A$
$\clubsuit 9$
$\diamondsuit 2$
$\heartsuit 2$
$\spadesuit K$

So, $\mathcal{A}$ gives $\heartsuit K$ to $\mathcal{C}$, and gives $\diamondsuit 8 - \heartsuit 7 - \heartsuit 4 - \heartsuit 2$ to $\mathcal{B}$.

$\mathcal{B}$ proceeds as follows:

1: Highest card $= \heartsuit 7$. $h = 33$.
2: The order is *adcb*. So, $n = 6$.
3: $k = 33 + 6 = 39$.
5: So the missing card is $\heartsuit K$.

4.6.5 Test case 3

$\mathcal{A}$ proceeds as follows:

1: Numbers for the cards, arranged in ascending order, are $x_1 = 1, x_2 = 9, x_3 = 15, x_4 = 27$ and $x_5 = 52$.
2: $d = 51 - 27 = 25$.
3: False
7: Give $x_1(\clubsuit A)$ to $\mathcal{C}$.
8: $k = 52 + 1 - 52 = 1$.
9: The order for 1 is *abcd*. So the order is $\clubsuit 9 - \diamondsuit 2 - \heartsuit 1 - \spadesuit K$.

So, $\mathcal{A}$ gives $\clubsuit A$ to $\mathcal{C}$, and gives $\clubsuit 9 - \diamondsuit 2 - \heartsuit 1 - \spadesuit K$ to $\mathcal{B}$.

$\clubsuit A$
$\clubsuit 9$
$\diamondsuit 2$
$\heartsuit 2$
$\spadesuit K$

$\mathcal{B}$ proceeds as follows:

1: Highest card = $\spadesuit K$. $h = 52$.
2: The order is *abcd*. So, $n = 1$.
3: $k = 52 + 1 - 52 = 1$.
5: So the missing card is $\clubsuit A$.

4.6.6 Test case 4

$\mathcal{A}$ proceeds as follows:

1: Numbers for the cards, arranged in ascending order, are $x_1 = 1, x_2 = 9, x_3 = 15, x_4 = 28$ and $x_5 = 52$.
2: $d = 51 - 28 = 24$.
3: True
4: Give $x_5(\spadesuit K)$ to $\mathcal{C}$.
5: The order for 24 is *dacb*. So the order is $\heartsuit 2 - \clubsuit A - \diamondsuit 2 - \clubsuit 9$.

$\clubsuit A$
$\clubsuit 9$
$\diamondsuit 2$
$\heartsuit 2$
$\spadesuit K$

So, $\mathcal{A}$ gives $\spadesuit K$ to $\mathcal{C}$, and gives $\heartsuit 2 - \clubsuit A - \diamondsuit 2 - \clubsuit 9$ to $\mathcal{B}$.

$\mathcal{B}$ proceeds as follows:

1: Highest card = $\heartsuit 2$. $h = 28$.
2: The order is *dacb*. So, $n = 24$.
3: $k = 28 + 24 = 52$.
5: So the missing card is $\spadesuit K$.

4.7 General Problem and Solution

4.7.1 Problem

Consider an ordered set of distinct n objects. $r + 1$ of these objects are chosen at random and given to $\mathcal{A}$. $\mathcal{A}$ selects one and gives that to $\mathcal{C}$, and hands over the remaining r objects to $\mathcal{B}$. From that, $\mathcal{B}$ recognizes the object given to $\mathcal{C}$.

Given r, how will you find the maximum n with which this trick can be done.

4.7.2 Solution

Using the second technique given above, r objects can be permuted into $r!$ ways, and using the concept of circular difference $2r!$ objects can be represented, and altogether $2r! + r$ objects can be used. So,

$$n = 2r! + r, r > 0$$

Let us check this for a few cases.

1. $r = 1 \Rightarrow n = 3$. This means you select two out of three objects at random, $\mathcal{A}$ gives one to $\mathcal{C}$ and the other to $\mathcal{B}$. $\mathcal{A}$ can do this in such a way that the object given to $\mathcal{C}$ is circularly next one from the object given to $\mathcal{B}$, so that $\mathcal{B}$ can immediately find out what it is. This cannot be done if there are four objects initially.
2. $r = 2 \Rightarrow n = 6$. This means, three out of six are selected at random, $\mathcal{A}$ gives one to $\mathcal{C}$, and two to $\mathcal{B}$. Two objects can be permuted to represent 1 or 2. This number, along with using difference in either direction, can represent any of the four remaining numbers.
3. $r = 3 \Rightarrow n = 15$.
4. $r = 4 \Rightarrow n = 52$. Somebody might have found that this number corresponds to the number of cards in a deck, and composed this puzzle. Since cards are divided into 13 ranks and four suits, it gave the other solution also.

4.8 Comments

Both techniques make use of the concept of modular (circular) arithmetic. The second technique is a general one. The first one is essentially the same as the second one, with the modular arithmetic reduced to one level down. This can be used in other cases also. For exanmple, when $r = 3, n = 15$, $\mathcal{A}$ can divide the object number by 3 and the quotient can be used in place of the suit in the original problem. He can find two objects with the same quotient. give one to $\mathcal{C}$, and put the other one as the first object to $\mathcal{B}$, and arrange the remaining 2 objects to show the difference from that object.

Practically, the first method is easier to be performed by magicians, because they need to memorize only a table of 6 permutations, rather than 24 in the second case.

Chapter 5

The puzzle of a door and four bolts

This puzzle was posted by **Tharn Jaggar** in *rec.puzzles* news group, and claimed that it is an original problem. Some solvers found that the idea was stolen from an old puzzle. (See below.) However, since this puzzle is better presented than the old one, I am giving this:

5.1 Question

You are in a room. On a wall there are two doors. The doors are locked *outside* by 4 horizontal iron bars (called bar1, bar2, bar3 and bar4).

A bar can slide from left to right and right to left between the two doors. A bar always locks one of the two doors. (check ASCII pic below). As they are outside the room, you have no clue of their position.

You can control the bars with buttons. When a bar is activated, it slides from its current door to the other one.

There are 3 buttons (called $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$)

1. Button $\mathcal{A}$ activates a *randomly choosen* bar:
   bar1 OR bar2 OR bar3 OR bar4
2. Button $\mathcal{B}$ activates *randomly* a choice amongst these:
   (bar1 AND bar2) OR (bar2 AND bar3) OR (bar3 AND bar4) OR (bar4 AND bar1)
3. Button $\mathcal{C}$ activates *randomly* a choice amongst these:
   (bar1 AND bar3) OR (bar2 AND bar4))

The problem is:

What is the shortest sequence of button you have to press to be sure you'll be freed ? (that is, to have all bars on either door1 or door2 at some point of the sequence)

ASCII pic: (this is just illustrative, it does not mean the initial configuration is this one)

```
    door1         door2
  ---------     ---------
 |         |   |         |
 |         |   |         |
 |         | <====bar1====>
<====bar2====> |         |
<====bar3====> |         |
 |         | <====bar4====>
 |         |   |         |
_|_________|___|_________|_______
```

5.2 Solution

Look at button $\mathcal{C}$. It will open a door once we reach the configuration 13/24 or 24/13, irrespective of which move of the two is done. That means, if we can find a way to reach 13/24 or 24/13, we can escape.

Also, after applying $\mathcal{B}$, 12/34 (or 34/12), the only other 2-2 combination, becomes /1234 after (12), 1234/ after (34), unlocking a door; whereas it becomes 13/24 after (23), or 24/13 after (41), which will be solved by applying $\mathcal{C}$ as above.

So, will $\mathcal{BC}$ unlock every 2-2 configuration? No, it won't if it is already in 13/24 form. We need to do $\mathcal{C}$ first, and if it didn't unlock, try $\mathcal{BC}$.

So, the sequence "$\mathcal{CBC}$" will unlock any 2-2 configuration.

Now, the remaining configuration is 1-3 (or 3-1) one. We can see applying $\mathcal{A}$, it will become either a 0-4 (or 4-0), which unlocks, or 2-2, which can be unlocked by applying "$\mathcal{CBC}$".

So, the strategy is

1. First try "$\mathcal{CBC}$", it will unclock if you had a 2/2 configuration originally.
2. If it didn't unlock, apply "$\mathcal{A}$", (now you have unlocked, or have 2-2) and then apply "$\mathcal{CBC}$" again.

So, "$\mathcal{CBCACBC}$" will unlock any initial configuration.

It seems to me, not very sure, that no other 7-move sequence will assure unlocking a door.

5.3 Solution 2

This solution* is essentially the previous one.

Assume the doors are not open- i.e. two bars are in different positions. WLOG, assume that 1 and 2 are in opposite positions. "$\mathcal{A}$" might simple flip bar #3 or #4 indefinintiely. Thus, you cannot be sure of escape on a press of button "$\mathcal{A}$".

Now, since two bars are in different positions, this requires that two *adjacent* bars are in different positons. Thus, any press of "$\mathcal{B}$" might flip those, and so cannot ensure the doors will be open on a press of "$\mathcal{B}$".

Obviously, the parity shift on adjacents given by "$\mathcal{C}$" is the key. I will break this into 3 cases (well, technically 4, but one case is that one door starts open, and so isn't interesting)

There are 3 cases:

1. the bars block alternately- say 1,2,1,2. Then pressing "$\mathcal{C}$" unlocks one door.

2. two consecutive bars block one door, the others block the secon- say 1,1,2,2. Pressing "$\mathcal{C}$" (to not ruin case 1, we need to start with "$\mathcal{C}$") will change this to either 1,2,21 or 2,1,1,2. Either of these is still this case. Pressing '$\mathcal{B}$' will give any of: 1,1,1,1; 2,2,2,2; 1,2,1,2; or 2,1,2,1. In any of these, we are out, or in case (1). Press "$\mathcal{C}$" to unlock a door in the latter position.

3. If one bar is different from the others (say 1,2,2,2 is the pattern), then pressing "$\mathcal{C}$" will yield 1,1,2,1 or 2,2,1,2. Pressing "$\mathcal{B}$" will yield 2,1,2,2; 1,1,1,2; 1,2,1,1; or 2,2,2,1. In all 6 of these possibilities, we are still in case (3) (again, we had to start with "$\mathcal{CBC}$" so we do not ruin cases 1 or 2). Pressing $\mathcal{A}$ will either give 2,2,2,2- and we are out; 1,1,2,2 or 1,2,21- and we are in case 2; or 1,2,1,2 and we are in case (1). Proceed as for a new problem, where case (3) is prohibited.

Thus, to ensure that one door is open at some point, we should press: $\mathcal{CBCACBC}$

1. In the case 1,2,1,2, a door is open on press #1.

*Due to Glen C. Rhodes.

2. In the case 1,1,2,2, a door is open on press #2 or #3, and NOT on press #1.

3. In the case 1,2,2,2, a door is not open on press #1,2, or 3, might be (but not necessarily be) open on #4, and then will be in one of the previous cases. Note that due to symmetry, 2,1,1,1 is the same as 1,2,2,2, and rotations of the bar numbers do not change the case.

5.4 Comments

John Francis, Derek Holt and Richard Forster pointed out that this puzzle is stolen from one of Martin Gardner's puzzles, which goes like this:

> You have a square table with a pocket at each corner (rather like a billiard table). In each pocket there is a cup.
>
> Before each move, while your back is turned, the table is rotated by some number of right angles. Then, you have the choice of one of the following moves:
>
> 1. Invert the cup in a single pocket.
> 2. Invert the cups in two adjacent pockets
> 3. Invert the cups in two diagonally-opposite pockets
>
> (you must nominate your move before reaching into any pocket)
>
> The goal is to get all the cups the same way up.

Chapter 6

Cups and genie (IBM puzzle July 2004)

This is IBM's *Ponder this* puzzle for July 2004[1]. This is the introduction from IBM's page:

> Ponder This Challenge: Puzzle for July 2004.
>
> This month's puzzle comes from Aditya K Prasad, who heard it from a friend. It is similar to our November 2002 puzzle (from Martin Gardner's February 1979 Scientific American column), but with an important difference. To be considered for publication, please supply answers for BOTH parts of the puzzle.

6.1 Question

Part 1: There is a round table divided into 4 equal quadrants, with one cup in each quadrant. The quadrants are labeled with letters (A, B, C, D) that do not move. Initially, each cup is randomly face-up or face-down. You are blindfolded and put in front of the table. On each turn of the game, you instruct a genie to flip the cups in whichever positions you choose (e.g., you may say "flip the cups in A and B"), possibly choosing no cups or possibly all four. The genie complies. At this point, if all the cups on the table are face-up, the genie will tell you that you have won the game and are free to go. If not, he rotates the cups randomly (possibly not rotating them) and you play another turn. Give a strategy to win this game in a finite number of moves (the solution is not unique).

Remark: the outcome of "rotation" the four cups is one of the four possible positions: the cups originally at (A,B,C,D) can be at (A,B,C,D), (B,C,D,A), (C,D,A,B), or (D,A,B,C). It is not an arbitrary permutation.

Notice that you cannot examine the current orientation of any cup at any time. This contrasts with the earlier puzzle.

Part 2: Suppose that instead of 4, there are n divisions and cups. For which n is it possible to guarantee a win? Prove your answer is correct.

6.2 Solution

The solution is possible exactly when n is a power of 2.

Suppose n is not a power of 2, so $n = r2^s$ with r odd and r at least 3.

Label the cups 0 through $n-1$. Also, *face-up* and *face-down* will be called values 0 and 1, respectively. Let the two "special" cups 0 and 2^s start out with opposite orientations. In the requested move, consider the positions with labels $(i \cdot 2^s, i = 0, 1, \cdots, r-1)$. Since r is odd, the requests (move, non-move) among these r positions cannot strictly alternate: there are two requests, separated by exactly 2^s, which agree (either both are to move, or both are not to move). Imagine that the cups have been rotated before fulfilling this request, so that our two special cups (initially at 0 and 2^r) fall into these two positions. Then after the move, these two cups still have opposite orientations. This can go on forever, no matter what we request.

Suppose instead that n is a power of 2. We will use induction to create a sequence of $2^n - 1$ requests which will satisfy the problem.

If $n = 1$, one move suffices: (0). (That is, we have a single request, which is to flip the single cup at position 0.) Either the cup is face up before the move, or it is afterwards.

Now suppose $n = 2 \cdot m$, where m is a power of 2. Let $S = (s_1, s_2, \cdots, s_{2^m-1)})$ be a sequence which works for m, where each s_i is a subset of $0, 1, \cdots, m-1$. Construct a sequence $T = (t_1, t_2, ..., t_{2^n-1})$ with t_i in $0, 1, ..., n-1$. The move $t_(i \cdot 2^m)$ is s_i (it only operates on the first m elements); it is called an *outer move*. The move $t_(i \cdot 2^m + j), j = 1, 2, ..., (2^m) - 1$, is $(s_j, m + s_j)$. That is, operates on first m elements the same way s_j did, and also acts identically on the second m elements. These *inner moves* ($t_{i \cdot 2^m + j}, j$ nonzero) leave the relation between items x and $m+x$ the same: either they remain identical or they remain opposite each other. Only the *outer moves* ($t_{i \cdot 2^m}$) change these relations. Keeping track of the relations, $r_i = (x_i + x_{m_i}) mod 2, i = 0, 1, \cdots, m-1$, there are only m of them; one of the $(2^m) - 1$ outer moves (say $t_{i \cdot 2^m}$) brings all these relations to all 0, by induction. Then during the block of inner moves between $i \cdot 2^m$ and $(i+1)2^m$, we have always $x_k = x_{m+k}$, and the inner moves bring all the $x_k (k < m)$ to 0, simultaneously bringing the x_{m+k} also to 0.

For $n = 2$, the complete set of 3 moves is $(0, 1), (0), (0, 1)$.

For n=4, the complete set of 15 moves is

```
(0,1,2,3), (0,2), (0,1,2,3), (0,1),
(0,1,2,3), (0,2), (0,1,2,3), (0),
(0,1,2,3), (0,2), (0,1,2,3), (0,1),
(0,1,2,3), (0,2), (0,1,2,3).
```

For $n = 8$ there are 255 moves.

The number of moves, $2^n - 1$, is clearly optimal: we get exactly 2^n chances to win (counting the initial position), and if the cups were initially random, each time we would have a chance $1/2^n$ of winning.

Chapter 7

The puzzle of Three Saints

7.1 Question

There are three identical-looking, infinitely wise and knowledgeable saints - they know everything, but speak only a little. Well, only two words – *wang* and *woong*. One of this means "Yes" and the other "No", which one is which you don't know.

One of the saints tells only truth. (he tells the word meaning "yes" when the answer is "yes" and the word meaning "no" when the answer is "no".)

Another tells only lies. (he tells the word meaning "no" when the answer is "yes" and the word meaning "yes" when the answer is "no".)

The third one has gone insane during these years, and irrespective of the question asked, he says *wang* or *woong* randomly in answer to any question.

You don't know which saint is which. That is what you need to find.

You can ask the saints three questions altogether. Not necessarily one per each saint. You can ask all the questions to one of them, or you can ask one each, or you can ask one question to one and two questions to another. All questions should result into an "Yes/No" answer. If you ask the same question to two saints, that is counted as two questions. You can decide the next question and the saint to ask depending on the answer of the previous question.

How do you do that?

Note: You need not find which of *wang* and *woong* is "Yes" and which is "No". You need to find which saint is True, False and Random.

7.2 Solution

7.2.1 First question

The goal of this question is to identify one saint who is not random. We are going to ask a question to $\mathcal{A}$ and identify one among $\mathcal{B}$ and $\mathcal{C}$ who is not random.

Ask $\mathcal{A}$: *Is the value of* $\mathcal{B}$ ***is Random XOR you are True XOR "wang" means "yes"*** *true?*

Here are the possibilities:

A	B	C	wang/woong	Result	Answer
T	F	R	Y/N	F xor T xor T = T xor T = F	woong
T	F	R	N/Y	F xor T xor F = T xor F = T	woong
T	R	F	Y/N	T xor T xor T = F xor T = T	wang
T	R	F	N/Y	T xor T xor F = F xor F = F	wang
F	T	R	Y/N	not (F xor F xor T) = not (F xor T) = not T = F	woong
F	T	R	N/Y	not (F xor F xor F) = not (F xor F) = not F = T	woong
F	R	T	Y/N	not (T xor F xor T) = not (T xor T) = not F = T	wang
F	R	T	N/Y	not (T xor F xor F) = not (T xor F) = not T = F	wang

If the answer is **wang**, $\mathcal{C}$ is guaranteed to be not random. If the answer is **woong**, $\mathcal{B}$ is guaranteed to be not random. This applies to the case when $\mathcal{A}$ is random, because in that case neither $\mathcal{B}$ nor $\mathcal{C}$ is random.

So, we identified a saint who is not random (either T or F). We'll ask the remaining two questions to him.

7.2.2 Second question

The goal of this question is to identify whether the *non-random* saint we identified tells is True or False.

Ask the non-random saint : *Is the value of* $2+2=4$ ***XOR "wang" means "yes"*** *true?*

Saint	wang/woong	Result	Answer
T	Y/N	T xor T = F	woong
T	N/Y	T xor F = T	woong
F	Y/N	not (T xor T) = not F = T	wang
F	N/Y	not (T xor F) = not T = F	wang

So, if the answer is "woong", he is True. If the answer is "wang", he is False.

7.2.3 Third question

Now we identified the (non-random) identity of one of the saints. We can ask him about another saint, and deduce the third one.

Ask the non-random saint : *Is the value of* ***$\mathcal{A}$ is Random XOR "wang" means "yes"*** *true?*

This is obvious. If the answer is "wang", A is random, and the third saint belongs to the third category. If the answer is "woong", the third saint is random, and A belongs to the third category.

Chapter 8

Four jealous husbands

This popular puzzle* has a lot of variations and versions.

8.1 Question

It is told that four men eloped with their sweethearts, but in carrying out their plan were compelled to cross a stream in a boat which would hold but two persons at a time. In the middle of the stream, there is a small island.

1. It appears that the young men were so extremely jealous that not one of them would permit his prospective bride to remain at any time in the company of any other man or men unless he was also present.

2. Nor was any man to get into a boat alone when there happened to be a girl alone, on the island or shore, other than the one to whom he was engaged.

Let us suppose the river to be two hundred yards wide, with an island in the middle on which any number can stand. How many trips would the boat make to get the four couples safely across in accordance with the imposed conditions?

8.2 Solution

Let the four husbands are `A`, `B`, `C` and `D`; and their respective wives `a`, `b`, `c` and `d`. Let us consider smaller cases first:

*Posted by Sharad Kedia in *World of Puzzles* yahoo group.

8.2.1 One couple

Too trivial.

8.2.2 Two couples

It is pretty easy. Get the wives out of the left shore. Then get the husbands to the right shore, and the wives to the left. Then the wives can manage to help others to cross to the right shore.*

#	Direction	Passengers	Left shore	Right shore
1	$L \Rightarrow R$	a b	A B	a b
2	$L \Leftarrow R$	b	A Bb	a
3	$L \Rightarrow R$	A B	b	Aa B
4	$L \Leftarrow R$	a	a b	A B
5	$L \Rightarrow R$	a b		Aa Bb

Table 8.1: Solution with two couples

8.2.3 Three couples

The principle above will work here also - Get all the wives to the right shore and all husbands still on the left shore, get all the husbands on the right shore and all the wives on the left shore by using the right wives to bring the right husband, and then the wives managing to help one another to cross the river.

*There is another way to get this done, without the strategy discussed. It is:

#	Direction	Passengers	Left shore	Right shore
1	$L \Rightarrow R$	Aa	Bb	Aa
2	$L \Leftarrow R$	A	A Bb	a
3	$L \Rightarrow R$	A B	b	Aa B
4	$L \Leftarrow R$	a	ab	A B
5	$L \Rightarrow R$	a b		Aa Bb

#	Direction	Passengers	Left shore	Right shore
1	$L \Rightarrow R$	a b	A B Cc	a b
2	$L \Leftarrow R$	b	A Bb Cc	a
3	$L \Rightarrow R$	b c	A B C	a b c
4	$L \Leftarrow R$	a	Aa B C	b c
5	$L \Rightarrow R$	B C	Aa	Bb Cc
6	$L \Leftarrow R$	Bb	Aa Bb	Cc
7	$L \Rightarrow R$	A B	a b	A B Cc
8	$L \Leftarrow R$	c	a b c	A B C
9	$L \Rightarrow R$	a c	b	Aa B Cc
10	$L \Leftarrow R$	c	b c	Aa B C
11	$L \Rightarrow R$	b c		Aa Bb Cc

Table 8.2: Solution with three couples

8.2.4 Four couples

One can see that the maximum number of couples that can be transported this way is three. In the solution for the three-couple puzzle above, we can see that one or more wives are kept alone (without their husbands) on some shore (in steps 1–4 and 7–11. Adding one more couple will beat this solution, because that couple cannot be left with them on any shore.

But the puzzle we are required to solve allows one more place to keep the people - an intermediate island. This provides more options to “mix and match” husbands and wives.

It is quite obvious that before the final phase, the husbands will be in the right shore, after which the wives can help themselves bringing the others. This is obvious from the 3-couple case.

It is also obvious that we can get all the wives easily to another place. What is difficult is the phase where the husbands are transported to the right shore.

So, we can use this plan:

1. Get all the wives from the left shore to the island.

2. Get all the husbands from the left shore to the right shore.

3. Get all the wives from the island to the right shore.

Here is how it is executed:

Get the four wives to the island

This is quite simple. Since all husbands are on the left shore together, the wives will not have any problem in getting transported to the island, by going two at a time, and one returning the left shore.

#	Direction	Passengers	Left shore	Island	Right shore
1	$L \rightarrow I$	a b	A B Cc Dd	ab	
2	$L \leftarrow I$	b	A Bb Cc Dd	a	
3	$L \rightarrow I$	bc	A B C Dd	a b c	
4	$L \leftarrow I$	b	A Bb C Dd	a c	
5	$L \rightarrow I$	bd	A B C D	a b c d	

Table 8.3: Four couples : Step 1 : Get all wives to the island

Get the four husbands to the right shore

This can be done by one wife going to the left shore, picking her husband from the left shore and dropping him on the right shore, and returning to the island. This can be repeated by each wife, thereby moving all the husbands from left shore to the right shore.

#	Direction	Passengers	Left shore	Island	Right shore
6	$L \leftarrow I$	a	Aa B C D	a c d	
7	$L \Rightarrow R$	Aa	B C D	b c d	Aa
8	$I \leftarrow R$	a	B C D	a b c d	A
9	$L \leftarrow I$	b	Bb C D	a c d	A
10	$L \Rightarrow I$	Bb	C D	a c d	A Bb
11	$I \leftarrow R$	b	C D	a b c d	A B
12	$L \leftarrow I$	c	Cc D	a b d	A B
13	$L \Rightarrow R$	Cc	D	a b d	A B Cc
14	$I \leftarrow R$	c	D	a b c d	A B C
15	$L \leftarrow I$	d	Dd	a b c	A B C Dd
16	$L \Rightarrow I$	Dd		a b c	A B C Dd
17	$I \leftarrow R$	d		a b c d	A B C D

Table 8.4: Four couples : Step 2 : Get all husbands to the right shore

Get the four wives to the right shore

Now since all the husbands are together on the right shore, there ia no question of jelousy and the wives can help themselves crossing the river.

#	Direction	Passengers	Left shore	Island	Right shore
18	$I \rightarrow R$	a b		c d	Aa Bb C D
19	$I \leftarrow R$	b		b c d	Aa B C D
20	$I \rightarrow R$	b c		d	Aa Bb Cc D
21	$I \leftarrow R$	c		c d	Aa Bb C D
22	$I \rightarrow R$	c d			Aa Bb Cc Dd

Table 8.5: Four couples : Step 3 : Get all wives from the island to the right shore

8.3 General solution

It can be seen that, if an intermediate stop is allowed, *any* number of couple can be transported this way. If there are n couples, they can cross the river in $(7n - 6)$ steps by

1. Transferring the n wives to the island in $(2n-3)$ steps (Transferring $(n-2)$ wives in $2(n-2)$ steps and the last two wives rowing together in one step),
2. Transferring the n husbands in $3n$ steps (A wife going back to the left shore, husband and wife rowing to the right shore, and the wife returning to the island), and
3. Transferring the n wives to the right shore in $(2n - 3)$ steps, as in Case 1.

So, the total number of rowings required is

$$2n - 3 + 3n + 2n - 3 = 7n - 6$$

8.4 Comments

I am afraid the solution given above is not the optimal one. The condition (2) in the question has no significance in this solution.

If somebody knows the correct solution, please let me know.

8.5 Obpuzzle

If the boat can accommodate m persons,

1. what is the maximum number of couples that can be transported from the left shore to the right shore?

2. what is the minimum number of trips needed to transport n couples?

Chapter 9

Puzzle of prisoners' strategy

IBM Research posts a reasonably tough puzzle every month at their *Ponder this* page[1]. This is the July 2002 puzzle.

9.1 Question

Ponder This Challenge:

This puzzle has been making the rounds of Hungarian mathematicians'parties.

The warden meets with the 23 prisoners when they arrive. He tells them:

1. You may meet together today and plan a strategy, but after today you will be in isolated cells and have no communication with one another.

2. There is in this prison a *switch room* which contains two light switches, labelled $\mathcal{A}$ and $\mathcal{B}$, each of which can be in the *on* or *off* position. I am not telling you their present positions. The switches are not connected to any appliance. After today, from time to time, whenever I feel so inclined, I will select one prisoner at random and escort him to the switch room, and this prisoner will select one of the two switches and reverse its position (e.g. if it was *on*, he will turn it *off*); the prisoner will then be led back to his cell. Nobody else will ever enter the switch room on that day.

3. Each prisoner will visit the switch room aribtrarily often. That is, for any N it is true that eventually each of you will visit the switch room at least N times.)

4. At any time, any of you may declare to me: *We have all visited the switch room.* If it is true (each of the 23 prisoners has visited the switch room at least once), then you will all be set free. If it is false (someone has not yet visited the switch room), you will all remain here forever, with no chance of parole.

Devise for the prisoners a strategy which will guarantee their release.

9.2 Solution

This solution was posted on the IBM site.

Elect a spokesman.

Each prisoner other than the spokesman maintains a counter with initial value 0. When he enters the switch room, if switch "A" is "off" and his counter is 0 or 1, then he switches "A" to "on" and increments his counter. Otherwise (switch "A" is already "on" or his counter is 2) he switches "B".

The spokesman also has a counter with initial value 0. When he enters the switch room, if switch "A" is "on", he switches "A" to "off" and increments his counter. Otherwise (switch "A" is already "off") he switches "B". When the spokesman's counter reaches 44, he declares to the warden "We have all visited the switch room."

He is safe in making this declaration: among the 44 times that the switch had been "on", at most once was because the switch might have started out in the "on" position at the beginning of time. At most two were due to each prisoner (other than the spokesman himself) turning it on. If not everyone had visited the switch room, then it could have been turned "on" at most 2*21=42 times, and his counter would not exceed 42+1=43.

Further, given enough time, each prisoner will have two opportunities to turn "on" the switch, so that the spokesman's counter will eventually reach or exceed 44.

Switch "B" is only used so that the prisoner has something to flip when the protocol says he should not flip switch "A".

The non-spokeman prisoners turn the switch on twice instead of just once, because of the uncertainty about its initial position.

9.3 Solution 2

This solution* was posted to *rec.puzzles* newsgroup.

*Due to Mark Pilloff.

When the players meet, one of them is designated the leader and the other 22 are designated followers. Each player retains a local variable called count which starts at 0 and will be incremented according to the strategy below. In general, switch A will be the switch with ”meaning” and switch B will simply be ignored except that it allows a player to flip a switch without distrubing the state of A.

The leader's strategy when entering the room:

1. If (A == on), toggle B.
2. If (A == off), A = ON and count++.
3. If (count == 44), declare that all players have been to the switch room.

Each followers strategy when entering the room:

1. If A == ON and count ¡ 2, A = OFF and count++,
2. Else toggle B.

Why does this work? The idea is that the followers only turn $\mathcal{A}$ OFF and each does so at most twice. Meanwhile the leader only turns $\mathcal{A}$ ON and is the only one to do so. The first time the leader does this, it is possible that she is the first to visit the room and $\mathcal{A}$ was initially set to the OFF position. But the remaining 43 times she can be certain that one of the followers has set the switch to OFF. Since no follower does this more than 2 times, all 22 followers must have done so at least once (since $2 \times 21 < 43$). Finally, by the *arbitrarily often* stipulation in the puzzle statement, it is clear that the end condition will eventually be met.

9.4 Further comments

I didn't solve the puzzle in the timeframe, but would like to give an analysis on solving the problem.

Consider a small variation of this puzzle, where it is given that both the switches are in the ”OFF” position initially. This makes a lot of difference. The prisoners can do this then.

1. Elect a leader. He keeps a counter. Also designate one of the switches (whose intial state is ”OFF”) as the ”main switch” and the other as ”dummy switch”.
2. Whenever the leader enters the room,
   (a) If the main switch is ”ON” (means no new prisoner has entered the room since his last visit), he flips the dummy switch.

(b) if he sees the main switch in "OFF" position, he turns it on, and increments the counter.

For the first time, it has been "OFF" till then, so he counts himself. From the next time onwards, it is "OFF" because some other **new** prisoner has turned it "OFF"(See below for others' strategy.), so he increments the counter, counting the new prisoner.

3. When a prisoner other than the leader enters the room,

   (a) If he finds the main switch in the "OFF" position (which means the leader is yet to count one of the other prisoners), he just flips the dummy switch.

   (b) If he finds that the main switch is in the "ON" position, switch it "OFF". This communicates to the leader that he has entered the room. But do this **only once** - in the subsequent cases, even if he finds that in the "ON" position (to prevent the leader from counting him twice), he flips the dummy switch.

When the leader has counted all the 23 prisoners, i.e., when the count reaches 23, he can declare that everyone has visited the room.

This is possible if they know the initial state of the main switch. (If it is the "ON" position initially, reverse ON/OFF in the above strategy.) If they do not know the initial state, they need to make sure that the count works irrespective of the initial state.

Now, let us say they adopt the same strategy. It will work fine if the main switch is "OFF" initially. But if it is already "ON", the first prisoner switches it OFF. The leader cannot count him because he doesn't know whether it was initially OFF or some prisoner switched it off, so he doesn't count it. The prisoner, who doesn't know whether the leader has visiited the room or not, will never do it again, and the leader's count will remain on 22 for ever. They can never get out.

So, a modification of the strategy is required. Each prisoner, other than leader, instead of once, flips the main switch from "ON" to "OFF" **twice**. So there will be a total of 44 flippings of "ON" to "OFF" of the main switch by the prisoners, and the leader will see at least 43 of them (because he only flips it to "ON" after the initial prisoner.) When he sees 43, he is sure that 21 of the remaining prisoners have flipped it twice, and one has flipped it either once or twice (he might have flipped once before the leader entered the room.). This is sufficient to declare that everyone has visited the room.

So, the revised strategy is:

1. Elect a leader. He keeps a counter. Also designate one of the switches (whose intial state is unknown) as the "main switch" and the other as "dummy switch".

2. Whenever the leader enters the room,
   (a) If the main switch is "ON", he flips the dummy switch.
   (b) if he sees the main switch in "OFF" position, he turns it on, and increments the counter.

       Here he doesn't count himself. He knows he was there. He counts the others. When he knows that all 22 have visited, he can make the declaration.
3. When a prisoner other than the leader enters the room,
   (a) If he finds the main switch in the "OFF" position (which means the leader is yet to count one of the other prisoners), he just flips the dummy switch.
   (b) If he finds that the main switch is in the "ON" position, switch it "OFF". This *may* communicate to the leader that he has entered the room. But do this **¡twice and only twice -** in the subsequent cases, even if he finds that in the "ON" position (to prevent the leader from counting him twice), he flips the dummy switch.

When the leader is sure that he has counted all the other 22 prisoners, i.e., when the count reaches 43, he can declare that everyone has visited the room.

Chapter 10

Mule and sugar

Steven Bytnar of Software Technology Group asked this puzzle in 2000. He said he picked it from some website.

10.1 Question

Your truck, carrying 10,000 lbs of sugar and a mule, breaks down. You must use the mule to carry the sugar the rest of the way, a 1,000 mile stretch of straight road, to your home. You can load sugar on to the mule to carry, but there are two caveats: a) the mule can't carry more than 1,000 lbs at any given time; and b) the mule will continuously eat the sugar he's carrying, at the rate of 1 lb per mile moved, whenever he's moving, and he will not move, in any direction, if he has no sugar to eat.

Assuming that sugar may only be transported by the mule (you can't carry any, the mule can't pull the truck, and you get no external help), and that you can't prevent the mule from eating the sugar, what is the most amount of the sugar, if any, that can make it to your house?

There are really two problems: deriving a method which yields the maximum amount of sugar at the end of the trip, and reasoning that no other method could yield more.

(*Hint:* At first blush, it might look like the answer is 0: If you put 1,000 lbs on the mule's back and walk straight home, the mule will have just emptied the bag (1 lb/mile x 1000 miles) when you walk in the door of your home.)

10.2 Answer

1399.7665904787 pounds.

10.3 Solution

My solution is based on the following considerations.

1. You always take 1000 lb from the starting point, as the mule eats sugar per mile, and not per pound. Also, you will try to load the mule with as much weight as you can while going, preferably 1000 pounds.
2. You leave some sugar on the way, so that you can use it on the way back and next time when you go.
3. When you go towards home, you will take enough sugar from each intermediate place to make the weight equal to 1000 pounds. On way back, you take sugar just enough to take you to the next location.
4. At the end, you won't have anything on the way. You will take home all sugar the mule didn't eat. This means, on the last trip, every intermediate place will have the exact amount of sugar to make the weight equal to 1000.

Now, Let us say, you stopped at x_1 miles (location L_1) for the first time and come back, a further x_2 miles (total $x_1 + x_2$ miles) on the second time (location L_2) etc. You need to do this 10 times to carry all the 10,000 pounds of sugar. So, you will have at most

$$x_1, x_2, x_3, \ldots x_{10}.$$

Some of these values may be zeros, if you have already covered 1000 miles.

Our aim is to find the values of $x_1, x_2, x_3, \ldots x_{10}$ for optimal solution.

Now, let us see.

1. On the first trip, you stop at x_1 miles, you have $(1000-x_1)$ pounds of sugar now. You will need x_1 pounds to go back. So, unload $(1000 - x_1 - x_1) = (1000 - 2x_1)$ pounds there.
2. On second trip, when you reach x_1 miles, you will have $(1000-x_1)$ pounds, takex_1 from there, go till $(x_1 + x_2)$ miles, at L_2.
3. Now you have,
    (a) 8000 pounds at the start

   (b) $(1000 - 3x_1)$ at L_1,
   (c) $(1000 - x_2)$ at L_2.

4. Leave $(1000 - x_2 - x_2)$ at L_2, and come back uptoL_1 with the x_2 pounds, take x_1 from L_1, and come back to the starting point. Now, L_1 has $(1000 - 4x_1)$ and L_2 has $(1000 - 2x_2)$.
5. On the third trip, take x_1 pounds from L_1 and x_2 pounds from L_2, go till $(x_1 + x_2 + x_3)$ miles (L_3).
6. Now you have,
   (a) 7000 pounds at the start,
   (b) $(1000 - 5x_1)$ at L_1,
   (c) $(1000 - 3x_1)$ at L_2,
   (d) $(1000 - x_3)$ at L_3.

Continuing this, after 10 such trips, you will have

1. $(1000 - 19x_1)$ at L_1
2. $(1000 - 17x_2)$ at L_2
3. $(1000 - 15x_3)$ at L_3
4. ...
5. ...
6. $(1000 - x_{10})$ at L_{10}

Ideally, you need all the intermediate sites to have zero pounds in the end, which means $x_1 = 1000/19, x_2 = 1000/17$ etc.

But now

1. L_1 is at $1000/19 = 52.63$
2. L_2 is at $1000(1/19 + 1/17) = 1000 * (36/323) = 111.46$
3. L_3 is at $1000(1/19 + 1/17 + 1/15) = 1000 * (863/4845) = 178.12$
4. L_4 is at $1000 * (16064/62985) = 255.04$
5. L_5 is at $1000 * (239689/692835) = 345.95$
6. L_6 is at $1000 * (2850036/6235515) = 457.07$
7. L_7 is at $1000 * (26185767/43648605) = 599.92$

8. L_8 is at $1000 * (174577440/218243025) = 799.92$

9. L_9 is at $1000 * (1396704420/654729075) = 1133.26$

We need not go this far. After L_8, we will straightaway go to 1000.

So, on the 9th trip, when you reach L_8, you will have 1000 pounds, as usual. The mule will eat 200.08, you will need another 200.08 to go back to L_8, leaving 599.84 pounds behind. To be precise, $1000-2\cdot1000(1-174577440/218243025) = 1000 - 2000(43665585/218243025) = 599.8443938000 < /B >< /tt >$ pounds.

On the 10th and last trip, you will go with the last 1000 pound bag, the sugar at every place will be just enough to make the weight equal to 1000 pounds, and when you take the sugar at L_8, you will have 1000 pounds.

The mule will eat 200.08 pounds at the remaining loop, leaving you 799.92 pounds.

So, in total, you have $599.84 + 799.92 = 1399.76$ approximately.

To be exact, it is

$$\begin{aligned}\text{Total} &= 1000 - 2000 \cdot \frac{43665585}{218243025} + 1000 - 1000 \cdot \frac{43665585}{218243025} \\ &= 2000 - 3000 \cdot \frac{43665585}{218243025} \\ &= 1399.7665904787 \text{pounds}.\end{aligned}$$

10.4 Solution 2

This is how most people solved this puzzle. This is prepared by me, using the methods described by Anita Raj and Sonal Ranjan.

I will transport ALL remaining sugar from one location to another location, making as many trips I need. Then I will transport ALL remaining sugar to the next location etc.

Now, if I have x pounds left, I need $\lceil \frac{x}{1000} \cdot 2 - 1 \rceil$ trips to transport it to the next location. So, to make it economical, x should be a multiple of 1000 always, or I will be wasting two trips.

Now, to make the remining sugar at every location a multiple of 1000, I should make sure that the mule eats only a multiple of 1000 amount of sugar between locations. In other words, the mule should walk only a integral multiple of 1000 miles between locations.

So, if x is the remaining sugar at a location, we need $\left(\frac{x}{1000} \cdot 2 - 1\right)$ trips, amounting a total of 1000n miles, where n is an integer. So, the next location should be at a distance of $\frac{1000n}{\frac{x}{1000} \cdot 2 - 1}$ miles.

I tried putting $n = 1$, which gave locations at (0.00, 52.63, 111.46, 178.12, 255.04, 345.95, 457.06, 599.92, 799.92) and yielded 1399.77, my original answer.

This corresponds to 1000/19, 1000/17 etc. and covering the last 200 miles differently.

Putting $n = 2$ gave locations at (0.00, 105.26, 238.60, 420.41, 706.13) and yielded 1118.39.

Putting $n > 2$ didn't help me to find any other solution.

So, I used a computer program. The result was...

Stops								**Yield**
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	
263.16	485.38	685.38						1056.14
263.16	374.27	517.13	717.13					1151.38
210.53	392.34	535.20	735.20					1205.60
210.53	301.44	412.55	555.40	755.40				1266.21
157.89	311.74	422.85	565.71	765.71				1297.13
157.89	234.82	325.73	436.84	579.70	779.70			1339.09
105.26	238.60	329.51	440.62	583.47	783.47			1350.42
105.26	171.93	248.85	339.76	450.87	593.73	793.73		1381.19
52.63	111.46	178.12	255.04	345.95	457.06	599.92	799.92	1399.77

10.5 Solution 3

This solution was provided by Steven Bytnar who asked this puzzle.

Thanks to all those who solved, and tried to solve, the most recent MRP Fun Quiz. I wish I could give a complete solution, but I can't; I don't yet have a full proof that the answer given below is optimal over all possible methods (although I am pretty certain that it is, and have a sketch of a proof). However, let me at least state what I do know about the problem.

1. The most sugar that I can get home is

$$\frac{1357730200}{969969} = 1399\frac{9}{969969} = 1399.766590478664782070354825 77\cdots$$

Chapter 11

Red and Green Disks

This is an easy puzzle* appearing very hard initially.

11.1 Question

You are travelling in the jungles of Africa, when you are caught by a tribe of barbarians. They allow you to choose between death or solving their great challenge.

You are blindfolded and taken to a room, where you are asked to kneel. You feel hundreds of circular discs lying in front of you. You are told that one side of each disc is painted red, and the other, green. There are exactly 129 discs that currently are red side up. You have to divide the discs into two groups, such that each group has the same number of discs showing the red colour. Obviously, no peeking allowed.

11.2 Solution

Take 129 disks one by one, flip them, and put them in a group, say group $\mathbb{A}$. Let all others be in the other group, say $\mathbb{B}$. These two groups will have the same number of the disks with red face up.

Let n among the 129 disks were originally with red face up. So, group $\mathbb{B}$ has $(129 - n)$ disks with red face up.

Group $\mathbb{A}$ originally had n disks with red face up, and $(129 - n)$ disks with green face up. Since you are flipping *all* disks in group $\mathbb{A}$, it will end up with $(129 - n)$

*Posted by Vamsi Reddy in *World of Puzzles* yahoo group.

disks with red face up and n disks with green face up. So, both the groups will have the same number – $(129 - n)$ – of disks with red face up. This is true for any value of n, from 0 to 129.

Chapter 12

Subtraction–a game

This puzzle* was originally published in the *Mindsport* column by Mukul Sharma in *Times of India*.

12.1 Question

In the subtraction of one three-digit number from another, Mary and Ann fill in the six digits as follows: Ann chooses a number from 0 through 9, and Mary chooses where to enter it. They continue thus until all blanks are filled. A number may be repeated, and leading zeroes are permitted. Ann tries to make the bottom line (the difference) the greatest and Mary, the smallest. If both players play their best what will be the bottom line?

12.2 Solution

If the number chosen by Ann is too low (< 4), Mary will put that in the hundredth column of the upper number. If it is too high (> 5), Mary will put that in the hundredth column of the lower number. So, Ann should choose 4 or 5.

Ann can choose either of this. Her strategy is this:

Choose 5. If Mary puts it on the lower hundredth place, give 9 as the subsequent numbers. The difference will never be less than 400. If mary doesn't fill the hundredth place, continue giving 5 until only the upper hundredth place remains. Then give 9. Difference is 400.

*Posted by Sharad Kedia in *World of Puzzles* yahoo group.

Choose 4. If Mary puts it on the upper hundredth place, give 0 as the subsequent numbers. The difference will never be less than 400. If mary doesn't fill the hundredth place, continue giving 4 until only the lower hundredth place remains. Then give 0. Difference is 400.

So, in either case, the result is 400.

Mary can fill the unit digits first, but that doesn't change the outcome.

If Ann chooses 4, Mary will put it in the unit column of the upper number. If Ann choses 5, Mary will put that in the unit column of the lower number. If Ann continues with 4 or 5, Mary will first fill the unit digits, then tens digits and then the hundreds digits.

It doesn't matter whether Ann chooses 4 or 5. In order to maximize the final result, she should give the same number for four times, until all the unit and tens digits are filled.

After the unit and tenth digits are filled, Ann has two choices, both leading to the same result.

1. Choose 5. Mary will put that in the hundredth place of the lower number. Now Ann can choose 9. Now the difference will be 400.

2. Choose 4. Mary will put that in the hundredth place of the lower number. Now Ann can choose 0. Now the difference will be 400.

Finally, consider the case where Mary first chooses a unit digit, say, and then choose a hundredth digit. This is in favor of Ann. For example, if Mary puts the first 5 at lower unit digit, and the next 5 at lower hundredth place, Ann will give all 9s subsequently, giving the difference $999 - 595 = 404$. In a similar case with 4s, the difference will be $404 - 000 = 404$. If Mary fills the tenth digit and then the hundredth digit, the difference will be at least 440 ($999 - 559$ or $440 - 000$). So Mary should not go that way.

So, if both of them play optimally, the answer will be 400.

Part III

Simple Arithmetic puzzles

Chapter 13

How deep is the hole?

This is a puzzle by J. A. H. Hunter.*.

13.1 Question

Tom was in the basement playing with the power drill he'd given his son for Christmas, when the the boy went down. *"Could you bore a 5-inch hole with that, dad?"* Ken asked, *"I mean diameter."*.

"Not with this," Tom told him, *"and it would be a big hole."*.

The boy nodded. *"That's what I thought,"* he said. *"But teacher asked how deep the hole would be if we it drilled it centrally through a ball thirteen inches in diameter."*.

Who'd want to do it anyway? But Ken's teacher must have meant a true spherical ball, so how deep would the hole be?

Please give the answer and the method to get the answer.

13.2 Solution

We have a circle of known diameter (d = 13 inches) and a chord of known length (l = 5 inches). So, using the famous formula we use to use spherometer (can be derived from from Pythagorus theorem, or the theorem governing two intersecting chords in a circle, products of the smaller line segments will be the same), the *height* of the circle from the chord is

*Posted by Peter Heichelheim in *rec.puzzles* newsgroup.

$$\begin{aligned} h &= \frac{d - \sqrt{(d-l)(d+l)}}{2} \\ &= \frac{13 - \sqrt{(13-5)(13+5)}}{2} \\ &= \frac{13-12}{2} \\ &= \frac{1}{2} \end{aligned}$$

This means half inch* will be "lost" on either side, making the depth of the hole $13 - 1 = 12$ **inches**.

13.3 Solution 2

This is another way† to state the solution.

Draw a 13-inch circle, and a line through the center of the circle. Draw two lines parallel to this line, at distances of 2.5 inch, simulating the hole you drilled. Connect the points where these parallel lines touch the circle. Also draw the diagonals for the rectangle you have now created. From this it can be seen that there is a rightangled triangle with shortest 'rightangle-side' 2.5 inch and 'non-rightangle-side 6.5 inch (remember that the diagonal for the rectangle is exactly the diameter of the sphere!). Therefore half the depth of the hole equals sqrt$(6.5^2 - 2.5^2) = 6$ inches, making the depth of the hole.......*(drum roll)* **12 inches**.

13.4 A little more general solution

Since to get the final answer, we need to double the height of the circle from the chord (the result of the above formula) and subtract from the diameter, it is easy to see that the general formula (for the answer) is

$$\sqrt{(d-l)(d+l)} = \sqrt{d^2 - l^2}$$

In this case, $d = 13, l = 5$, so the answer is 12.

*The negative value of the square root will give 12.5, the height from the other side.

†Due to Ramon Verbruggen.

Chapter 14

The bee and the trains

This solution is often quoted to show how stupid Mathematicians can be while analyzing simple problems.

14.1 Question

This puzzle is often asked with numeric values. I am asking the general question here.

Two trains enter opposite ends of a straight, single track tunnel of length L miles. They have uniform speeds u and v miles per hour. Sitting on the front of one of the trains is a genetically engineered bumble bee. The bee can see in the dark and flies w miles per hour. w is greater than u and v. As soon as the trains enter the tunnel, the bee starts flying back and forth between the front of the two trains. How far will the bee have traveled before the two trains crash to each other?

14.2 Answer

The bee would have traveled

$$L \cdot \frac{w}{u+v} \quad \text{miles.}$$

14.3 Solution

The story about mathematicians solving this puzzle by summing up infinite progressions is interesting. However, this has a very simple solution.

Relative speed between the two trains is $u + v$ MPH, and they will meet after $L/(u + v)$ hours. The bee would have travelled $L/(u + v) \cdot w$ miles by then.

Part IV

Puzzles on coins and weighing

Chapter 15

Finding the nature of coin in one weighing

This is a simple logical puzzle* that needs a little arithmetic.

15.1 Question

There are 101 coins of which exactly 50 coins are artificial. The weight of an artificial coin is just 1 gm. less than that of a real coin. There is a balance which is able to indicate the difference in weights between its two weighning plates. Suppose, you have chosen a coin arbitrarily from those 101 coins. Can you tell us whether that coin is real or artificial by weighing just once ?

15.2 Solution

The total weight of the 101 coins is 50 grams less than the true weight.

If the considered coin is good, the remaining lot contains 50 good and 50 bad coins, the total weight of which will be 50 grams less than what it should be. If we divide those 100 coins into two groups of 50 each, the division of bad coins in the two groups will be (0, 50), (1, 49), (2, 48), ..., (48, 2), (49, 1) or (50, 0), giving a difference of -50, -48, -46, ..., 46, 48, 50 grams respectively. In all these cases, the difference in weight of the two groups will be an even number of grams.

*Posted by Pinaki Chakrabarti in *MCC June 2002 puzzle contest*.

If the considered coin is bad, the remaining lot contains 51 good and 49 bad coins, the total weight of which will be 49 grams less than what it should be. If we divide those 100 coins into two groups of 50 each, the division of bad coins in the two groups will be (0, 49), (1, 48), (2, 47), ..., (47, 2), (48, 1) or (49, 0), giving a difference of -49, -47, -45, ..., 45, 47, 49 grams respectively. In all these cases, the difference in weight of the two groups will be an odd number of grams.

So, the solution is

1. Put any 50 of the remaining coins on the left pan, and the remaining 50 coins on the right pan.
2. Find the weight difference.
3. If the difference is an even number of grams (0 is considered even), the coin considered is a good one.
4. If the difference is an odd number of grams, the coin considered is a bad (artificial) one.

15.3 Solution 2

This solution* is essentially the same as the previous one but expressed in a mathematical way.

Let us assume that the weight of each real coin is w and that of an artificial coin is (w-1) [w is in gm.]

I have a real coin: This implies, we have 50 real coins and 50 artificial coins to weigh. The left plate of the balance will have 50 coins and let us assume that of those 50 coins, x are artificial. The right plate of the balance will then have 50 coins of which $(50 - x)$ coins are artificial and x are real $(0 \leq x \leq 50)$.

So, the weight of the left plate is,

$$W_l = x(w - 1) + (50 - x)w = 50w - x$$

And the weight of the right plate is,

$$W_r = (50 - x)(w - 1) + xw = 50w - 50 + x$$

Thus the balance will show,

*Due to Pinaki Chakrabarti.

$$W_d = |W_l - W_r| = |50 - 2x|$$

which is an even number.

I have an artificial coin: This implies, we have 51 real coins and 49 artificial coins to weigh. The left plate of the balance will have 50 coins and let us assume that of those 50 coins y are artificial. The right plate of the balance will then have 50 coins of which $(49 - y)$ coins are artificial and $(1 + y)$ are real $(0 \leq y \leq 49)$.

So, the weight of the left plate is,

$$W_l = y(w - 1) + (50 - y)w = 50w - y$$

And the weight of the right plate is,

$$W_r = (49 - y)(w - 1) + (1 + y)w = 50w - 49 + y$$

Thus the balance will show,

$$W_d = |W_l - W_r| = |49 - 2y|$$

which is an odd number.

The solution is the same as the one given in the previous solution.

Part V

Miscellaneous puzzles

Chapter 16

Arithmetic with General Knowledge

Carl G. posted this puzzle in rec.puzzles alt.brain.teasers newsgroups.

16.1 Question

1. Start with an unlucky number for a Friday.
2. Multiply by the gables on Hawthorn's house.
3. Add the number a stitch in time saves.
4. Add the number of blind mice.
5. Subtract the number of William Pne du Bois' balloons.
6. Add the number of wonders of the world.
7. Subtract the number of miles in Camptown's racetrack.
8. Subtract the number of strings on a violin.
9. Divide by the number of vertices on a regular hexahedron.
10. Add the number of the engine that ran on Chicago line.
11. Multiply by the number of gentlemen of Verona.
12. Add the atomic number of the element whose symbol is the 25th letter of the alphabet.
13. Divide by the number of hills of Rome.

14. Multiply by the number of railroads on a Monopoly board.
15. Add the number of easy pieces.
16. Add the number of chromosomes in a normal human muscle cell.
17. Multiply by the number of kittens that lost their mittens.
18. Add the number of acres in A. A. Milne's woods.
19. Multiply by the number of cities in Dickens' tale.
20. Subtract the number of degrees Fahrenheit at which Bradbury's books burn.
21. Add the number of Great Lakes.
22. Divide by the number of days of the condor in the title of Grady's book.
23. Subtract the number of blackbirds baked in a pie.
24. Multiply by the number of horsemen of the Apocalypse.
25. Divide by the number of men on a dead man's chest.
26. Add the number of a neutral PH.
27. Subtract the number of carbon atoms in a molecule of ethane.
28. Multiply by the number of heads on Lofting's Pushme-Pullyou.
29. Add the number of miles on the road sign to Juster's Digitopolis.
30. Subtract the number of dried orange pips in a Sherlock Holmes case.
31. Multiply by the number of the square at which Alice met Humpty Dumpty.
32. Divide by the number of witches in Macbeth.
33. Divide by the number of suits in a standard deck of cards.

What is the result?

It is also stated that the final result and all intermediate results are integers.

16.2 Solution

My solution is given below:

1. Start with an unlucky number for a Friday.
   13 *(”Friday the thirteenth”)*
2. Multiply by the gables on Hawthorn’s house
   13 * 7 = 91 *(”The house of the seven Gables”)*
3. Add the number a stitch in time saves.
   91 + 9 = 100 *(”A stitch in time saves nine”)*
4. Add the number of blind mice.
   100 + 3 = 103 *(”Three blind mice, see how they run...”)*
5. Subtract the number of William Pne du Bois’ balloons.
   103 - 21 = 82 *(”The twenty-one balloons”)*
6. Add the number of wonders of the world.
   82 + 7 = 89
7. Subtract the number of miles in Camptown’s racetrack.
   89 - 5 = 84

   > *The Camptown ladies sing this song*
   > *The Camp-town race-track five miles long*
   > *I come down through with my hat caved in*
   > *I go back home with a pocket full of tin...*

8. Subtract the number of strings on a violin.
   84 - 4 = 80
9. Divide by the number of vertices on a regular hexahedron.
   80 / 8 = 10 *(6 faces, 8 vertices, 12 edges)*
10. Add the number of the engine that ran on Chicago line.
    10 + 9 = 19

    > *Engine, engine, number nine,*
    > *Sliding down Chicago line;*
    > *When she’s polished she will shine,*
    > *Engine, engine, number nine.*

11. Multiply by the number of gentlemen of Verona.
    19 * 2 = 38 *(Shakespaere’s book ”Two gentlemen of Verona”)*

12. Add the atomic number of the element whose symbol is the 25th letter of the alphabet.

    38 + 39 = 77 *(Y ⇒ Yttrium ⇒ 39)*

13. Divide by the number of hills of Rome.

    77 / 7 = 11

14. Multiply by the number of railroads on a Monopoly board.

    11 * 4 = 44

15. Add the number of easy pieces.

    44 + 5 = 49 *("Five easy pieces")*

16. Add the number of chromosomes in a normal human muscle cell.

    49 + 46 = 95 *(2 * 23 = 46)*

17. Multiply by the number of kittens that lost their mittens.

    95 * 3 = 285

    Three little kittens,
    They lost their mittens,
    And they began to cry:
    "O mother dear,
    We very much fear,
    That we have lost our mittens.

18. Add the number of acres in A. A. Milne's woods.

    285 + 100 = 385

    (Winnie the Pooh's The Hundred Acre Woods Expotition)

19. Multiply by the number of cities in Dickens' tale.

    385 * 2 = 770 *("The tale of two Cities")*

20. Subtract the number of degrees Fahrenheit at which Bradbury's books burn.

    770 - 451 = 319

21. Add the number of Great Lakes.

    319 + 5 = 324

22. Divide by the number of days of the condor in the title of Grady's book.

    324 / 6 = 54 *("Six days of the condor")*

23. Subtract the number of blackbirds baked in a pie.

    54 - 24 = 30 *("Four and Twenty blackbirds pie")*

24. Multiply by the number of horsemen of the Apocalypse.
    30 * 4 = 120
25. Divide by the number of men on a dead man's chest.
    120 / 15 = 8 *(Both 15 and 16 are popular, only 15 divides 120)*
26. Add the number of a neutral PH.
    8 + 7 = 15
27. Subtract the number of carbon atoms in a molecule of ethane.
    15 - 2 = 13 *(CH3-CH3)*
28. Multiply by the number of heads on Lofting's Pushme-Pullyou.
    13 * 2 = 26 *("double-headed llama")*
29. Add the number of miles on the road sign to Juster's Digitopolis.
    26 + 5 = 31 *("The Phantom Tollbooth" by Norton Juster)*
30. Subtract the number of dried orange pips in a Sherlock Holmes case.
    31 - 5 = 26 *("The five orange pips")*
31. Multiply by the number of the square at which Alice met Humpty Dumpty.
    26 * 6 = 156 *(d6, a.k.a Q6, from White's side)*
32. Divide by the number of witches in Macbeth.
    156 / 3 = 52 *("Foul is fair...")*
33. Divide by the number of suits in a standard deck of cards.
    52 / 4 = 13
    (Tricky. One will be tempted to divide by 13 here...)
34. What's the answer?
    13.

We are back to the unlucky number !

Bibliography

[1] `http://www.research.ibm.com/ponder`. IBM Research–*Ponder this* puzzles.

Index